

Fig. 14-Diagrammatic sketch of $1000-\mathrm{Mc}$ amplifier of the folded-back type.
isolate the anode and cathode circuits, it does permit stable amplifier operation. The amplifier circuit shown in Fig. 14 has the anode circuit folded back over the cathode circuit. This type of construction permits the insertion of the tube into the open end of the circuit. Connections are made to the cathode and grid of the tube by means of contact fingers, while the anode ring of the tube is clamped firmly in the circuit. The anode circuit is a coaxial line operating in the $\frac{3}{4}$-wavelength mode and the cathode line operated in the $\frac{5}{4}$-wavelength mode. Radio-frequency power is fed into the amplifier by means of a capacitive probe inserted into the portion of the cathode line which extends below the anode line. Power is taken from the amplifier by means of a probe inserted into the anode line near the tube. For measure-


Fig. 15-Performance curve of $1000-\mathrm{Mc}$ amplifier of the folded-back type.
ment purposes, the power output was fed into a watercooled load through a double-stub tuner.

A performance curve of the amplifier is shown in Fig. 15. This curve shows dc power input, rf power output, apparent plate efficiency, and power gain of the amplifier as a function of rf driving power. The driving power was measured by means of a standing-wave-detector type of wattmeter inserted between the amplifier and driver.

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# The Tapered Phase-Shift Oscillator* 

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#### Abstract

Summary-The tapered phase-shift networks with three and four sections are investigated. It is shown that an oscillator incorporating one of these networks and a triode tube is useful throughout the au-dio-frequency range.


## Introduction

$\llbracket$T IS THE PURPOSE of this paper to consider a modification of the phase-shift oscillator ${ }^{1}$ in which the impedance of the sections of the phase-shift network is increased progressively along the network. The

[^0]modified circuit has been used as a low-frequency oscillator ${ }^{2}$; however, it is so useful throughout the entire au-dio-frequency range that further investigation seemed desirable.

In practice, using the tapered network, it is possible to obtain stable oscillation with one section of a dual triode such as the 6SN7 or 12AU7. This is an improvement over the normal phase-shift oscillator, which usually requires a pentode for sufficient amplification and a diode for output control. In some applications, this may permit the saving of one or more tubes.

[^1]
## The Phase-Shift Oscillator

The normal phase-shift oscillator is shown in Fig. 1(a). It consists of a one-tube amplifier with the plate connected back to the grid through an $R C$ network. The network sections can be made up of series $C$ and shunt $R$ elements, as shown, or vice versa. For oscillation to be maintained, it is required that the gain from grid to plate must be at least equal to the attenuation from plate to grid. The phase shift from plate to grid must be an integral multiple of $180^{\circ}$. These conditions can be met in an $R C$ network of the type shown with three or more sections. With three sections, the attenuation through the network is 29 , while with four sections it is 18.2. The attenuation $k$ is defined as the ratio $E_{1} / E_{2}$, Fig. 1(a).

Less attenuation can be obtained with the network of Fig. 1(b). Here the element impedances of succeeding sections have been obtained by multiplying the impedances of the previous section by the factor $a$. Less attenuation is obtained because the loading imposed by any one section on the previous section is smaller.

## The Tapered Phase-Shift Network

Calculation of the attenuation and phase shift through the network is best carried out with the aid of matrix algebra. ${ }^{3}$ In using this method, a pair of equations is obtained in the form

$$
\left\{\begin{array}{c}
E_{1}=A E_{2}+B I_{2}  \tag{1}\\
I_{1}=C E_{2}+D I_{2}
\end{array}\right.
$$

where $A, B, C$, and $D$ are coefficients associated with the network and $I_{1}$ and $I_{2}$ are the currents, respectively, entering the network at the left-hand side and leaving at the right-hand side, Fig. 1(a).

If the input impedance of the amplifier is very high, $I_{2}=0$, and $E_{1}=A E_{2}$. Therefore, $E_{1} / E_{2}=A=k$, the attenuation previously defined. For the network of Fig. 1(b),

$$
\begin{align*}
A= & {\left[1-\omega^{2} R^{2} C^{2}\left(3+\frac{2}{a}\right)\right] } \\
& +J\left[\omega R C\left(3+\frac{2}{a}+\frac{1}{a^{2}}\right)-\omega^{3} R^{3} C^{3}\right]=-k \tag{2}
\end{align*}
$$

The minus sign is attached because of the required $180^{\circ}$ phase shift. The attenuation must be a real quantity; therefore, the imaginary term in (2) can be equated to zero to obtain $\omega$. Thus,

$$
\begin{aligned}
& \omega R C\left(3+\frac{2}{a}+\frac{1}{a^{2}}\right)-\omega^{3} R^{3} C^{3}=0 . \\
& \omega=\frac{\sqrt{3+\frac{2}{a}+\frac{1}{a^{2}}}}{R C}=\frac{N_{3}}{R C}
\end{aligned}
$$

${ }^{3}$ E. A. Guillemin, "Communication Networks," vol. II, John Wiley and Sons, New York, N. Y., 1935; p. 145.
where

$$
N_{3}=\sqrt{3+\frac{2}{a}+\frac{1}{a^{2}}} .
$$

Substituting this value for $\omega$ in (2), $k$ can be obtained:

$$
k=\frac{8 a^{3}+12 a^{2}+7 a+2}{a^{3}}
$$


(a)

(b)

(c)

Fig. 1-Phase-shift oscillator.
If the resistors and capacitors are interchanged, an equally useful network results. Another pair of networks can be obtained by using four sections. Table I lists ex-

TABLE I

| Network | $k$ | $\omega$ |
| :---: | :---: | :---: |
| Three- <br> section <br> shunt $C$ | $\frac{8 a^{3}+12 a^{2}+7 a+2}{a^{3}}$ | $\frac{\sqrt{3+\frac{2}{a}+\frac{1}{a^{2}}}}{R C}=\frac{N_{3}}{R C}$ |
| Threesection shunt $R$ | $\frac{8 a^{3}+12 a^{2}+7 a+2}{a^{3}}$ | $\frac{1}{R C \sqrt{3+\frac{2}{a}+\frac{1}{a^{2}}}}=\frac{1}{R C N_{3}}$ |
| Foursection shunt $C$ | $\begin{aligned} & 64 a^{6}+192 a^{5}+260 a^{4} \\ & +214 a^{3}+109 a^{2}+44 a+8 \\ & \hline 16 a^{6}+24 a^{5}+9 a^{4} \end{aligned}$ | $\frac{\sqrt{\frac{4 a^{3}+3 a^{2}+2 a+1}{4 a^{3}+3 a^{2}}}}{R C}=\frac{N_{4}}{R C}$ |
| Foursection shunt $R$ | $\begin{aligned} & 64 a^{6}+192 a^{5}+260 a^{4} \\ & +214 a^{3}+109 a^{2}+44 a+8 \\ & \hline 16 a^{6}+24 a^{5}+9 a^{4} \end{aligned}$ | $\frac{1}{R C \sqrt{\frac{4 a^{3}+3 a^{2}+2 a+1}{4 a^{3}+3 a^{2}}}}=\frac{1}{R C N_{4}}$ |

pressions for $\omega$ and $k$ applicable to these four networks. It will be noted that the attenuation is not altered by interchanging the resistors and capacitors.

Fig. 2 is a plot of $k$ versus $a$. Thus it shows the relation between $a$ and the minimum amplifier gain for sustained oscillation. It can be seen that a comparatively small value of $a$ will give an enormous decrease in the attenuation through the network.

Fig. 3 is a plot of $N$ versus $a$. With the aid of this figure and Table I, the frequency for $180^{\circ}$ phase shift through any one of the four networks can be obtained. Or, conversely, having a desired frequency, $R$ and $C$ can
be calculated. It must be remembered that these results assume that the network is being driven by a source having zero impedance.

The dashed lines of Figs. 2 and 3 indicate measurements made on some of the networks. A reasonably close check was obtained with theory. Phase shift was measured with an oscilloscope; attenuation was checked by substitution with a calibrated voltage source.

## Practical Considerations

The choice of the type of network, number of sections, and value of $a$ will depend on the tube type and fre-


Fig. 2-Attenuation versus $a$.


Fig. 3-Coefficient $N$ versus $a$.
quency desired. In general, it is desirable to use the shunt$R$ network at frequencies below 1000 cps because this network provides dc isolation between the plate and grid, and thus additional circuit elements are not required. The shunt- $C$ circuit is valuable above 1000 cps because the input capacitance of the tube, which may be high in triodes because of Miller effect, ${ }^{4}$ can be made a part of the last capacitor of the network.

Frequency stability will not be considered here; it has been covered in part by references 1 and 2 . It should be stated, however, that there are at least three ways in which tube parameters affect frequency: (1) The plate resistance can change, affecting the equivalent generator impedance, and, therefore, the phase shift through the network. (2) The gain of the amplifier may change, altering the Miller-effect loading on the last section of the network. (3) If the bias changes, the level at which limiting occurs will change, which would affect the frequency.

The design of such an oscillator is probably best illustrated by an example: It was desired to construct an oscillator at 1000 cps , using one section of a 12AU7 dual triode. Reference to the tube characteristics showed that a gain of about 12 could be obtained with the load and bias resistors shown in Fig. 1(c) and with the cathode resistor by-passed. From Fig. 2 it can be seen that a four-section network with $a=1.4$ will give sufficiently low attenuation. To allow for variations in tubes and
which is about 1.5 megohms. Thus the other resistors are determined.
Table I shows that $\omega=1 / R C N_{4}$ for the four-section shunt- $R$ network. Referring to Fig. 3, with $a=3, N_{4}$ $=1.03$. The values of the capacitors can then be obtained. It should be noted that variations in any one circuit element will not have a very great effect on the over-all network; it is usually satisfactory to choose the nearest stock values.
The last shunt resistor of the network in Fig. 1(c) was returned to ground to provide a proper dc grid return, while the other resistors were returned to the cathode. In this way, it is possible to eliminate the cathode bypass capacitor and still obtain sufficient gain. The frequency will differ from the calculated value as a result of the change; the difference was only about 5 per cent in this case.

Fig. 4 shows the performance of the circuit for variations in plate supply voltage, heater supply voltage, and plate and heater supply voltage together. It can be seen that the effects of plate supply voltage $E_{b b}$ and heater supply voltage $E_{f f}$ are opposite in direction, so that, when the two are taken together, a very stable oscillator is obtained. Increasing $E_{b b}$ from 150 to 350 volts, and increasing $E_{f f}$ from 5 to 7 volts at the same time, increases the frequency by only 0.15 per cent. The output voltage $E_{p}$ is nearly proportional to $E_{b b}$ but is almost independent of $E_{f f}$ as long as oscillation is maintained.


Fig. 4-Performance of tapered phase-shift oscillator.
supply voltages, it was decided to take $a=3$. Using the shunt- $R$ network, the value of the last resistor is determined by the maximum allowable grid circuit resistance,

- J. M. Miller, Bureau of Standards Bulletin, no. 351, 1919.

The total distortion was 3 per cent with $E_{b b}=250$ and $E_{f f}=6.3$ volts.

Other oscillators have been constructed with the shunt- $R$ network at frequencies as low as $1 / 5 \mathrm{cps}$. The shunt- $C$ network has been used at frequencies up to 20 kc .


[^0]:    * Decimal classification: R355.914.31. Original manuscript received by the Institute, March 18, 1948.
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    ${ }^{1}$ E. L. Ginzton and L. M. Hollingsworth, "Phase-shift oscillators," Proc. I.R.E., vol. 29, pp. 43-49; February, 1941.

[^1]:    ${ }^{2}$ R. W. Johnson, "Extending the frequency range of the phaseshift oscillator," Proc. I.R.E., vol. 33, pp. 597-602; September, 1945.

