# Phase-Shift Oscillators*

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***Summary***—*This paper describes a type of resistance-capacitance-tuned oscillator which operates with a single tube. A three- or more mesh phase-shifting network is connected between the output and input of an amplifier tube. When the gain of the amplifier is adjusted either manually or by an automatic-volume-control circuit barely to maintain oscillation, almost pure sine-wave output is obtained. Variations in the basic circuit have been analyzed and design formulas are included in this paper. Experimental work verified theoretical expectations. A typical oscillator was found to have a distortion of 0.1 per cent at an output voltage of 20 volts.*

## Introduction

There are many uses for audio-frequency oscillators in research laboratories and in industry for the testing of communication and other types of equipment. A variety of oscillators are in use, most of which depend upon inductance-capacitance resonant circuits for tuning and discrimination against harmonics. Simple oscillators of the Hartley type are satisfactory for medium and high audio frequencies because the coils and condensers that are required are small and may be easily constructed to have low losses. At very low audio frequencies such circuits become impracticable because it is difficult to construct the required large inductances to reduce sufficiently the losses. The difficulty of obtaining low-frequency oscillations can be overcome by means of heterodyning two high-frequency oscillators. Such beat-frequency oscillators have been used extensively in the past and are quite satisfactory for most applications. However, they have some disadvantages; namely,

1. Frequency stability is poor, since a small change in the frequency of one oscillator produces a large percentage change in the heterodyne frequency. This is especially true when the heterodyne frequency is low.
2. In order to prevent synchronization at low frequencies, the oscillators must be well shielded. This adds extra weight and increases the cost of construction.
3. Calibration is not constant and must be checked often.

Resistance-capacitance-tuned oscillators employing negative and positive feedback circuits have been developed in recent years to overcome these fundamental difficulties.[1,2] In these circuits, negative feedback is introduced in a two-stage resistance-capacitance-coupled amplifier through a network equivalent to a Wien bridge. This eliminates negative feedback at a frequency determined by the resistance-capacitance components of the Wien bridge. Consequently, the amplifier acts in a manner similar to a resonant circuit. If positive feedback is introduced in this amplifier, oscillation will take place at the frequency determined by the constants of the Wien bridge. Such an oscillator can be made to work at extremely low as well as at high frequencies. It is in many respects equivalent in performance to a good beat-frequency oscillator; and, in addition, it is simple, light, inexpensive, and does not need a calibrating adjustment.

The resistance-capacitance-tuned oscillator seems to have been described first by Scott[1] and later by Terman, Buss, Cahill, and Hewlett.[2] The authors of the present paper had the circuit of a single-tube resistance-capacitance-coupled oscillator suggested to them by J. R. Woodyard of Stanford University. This oscillator circuit and variations developed by the authors as described in this paper, accomplish the same things as Scott's and Terman's, but in a different and simpler manner. Its performance is about the same as that of other resistance-capacitance-tuned oscillators, with possibly a better frequency stability. Investigations by the authors revealed the fact that this oscillator circuit was not new but had been described by Nichols[3] in 1921. Nichols also showed circuits in which the frequency of oscillation was determined by an inductance-resistance combination. While there are applications where inductance-resistance tuning may be superior to the capacitance-resistance tuning, the present authors feel that the general principles involved are so similar in the two cases that a discussion of such circuits is omitted from this paper.

## Basic Circuits

In order to produce self-sustaining oscillations in an amplifier, two conditions must be satisfied. First, the voltage introduced from the output of the amplifier to its input must be in phase with the input voltage; second, the over-all amplification of the network must be equal to, or greater than, unity. That is, if

$A$ = amplification parameter of the amplifier

$\beta$ = fraction of the output voltage of the amplifier introduced into the input of the amplifier

then,

$$A\beta \geqq 1 \tag{1}$$

this being a vector relationship.
Thus, if

$$\begin{aligned} A &= |A| \underline{/\theta} \\ \beta &= |\beta| \underline{/\phi} \end{aligned} \tag{2}$$

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[1] H. H. Scott, "A new type of selective circuit and some applications," PROC. I.R.E., vol. 26, pp. 226–235; February, 1938.

[2] F. E. Terman, R. R. Buss, W. R. Hewlett, and F. C. Cahill, "Some applications of negative feedback with particular reference to laboratory equipment," PROC. I.R.E., vol. 27, pp. 649–655; October, 1939.

[3] H. W. Nichols, United States Patent 1,442,781, January 16, 1923. (Filed July 7, 1921.)

then

$$|A||\beta| \geqq 1 \tag{3}$$

$$\theta + \phi = 0. \tag{4}$$

In other words, an oscillator must consist of an amplifier capable of developing a voltage amplification $1/|\beta|$ and a phase-shifting network which will satisfy (4).

Each one of the circuits shown in Fig. 1 is composed of a one-tube resistance-capacitance-coupled amplifier

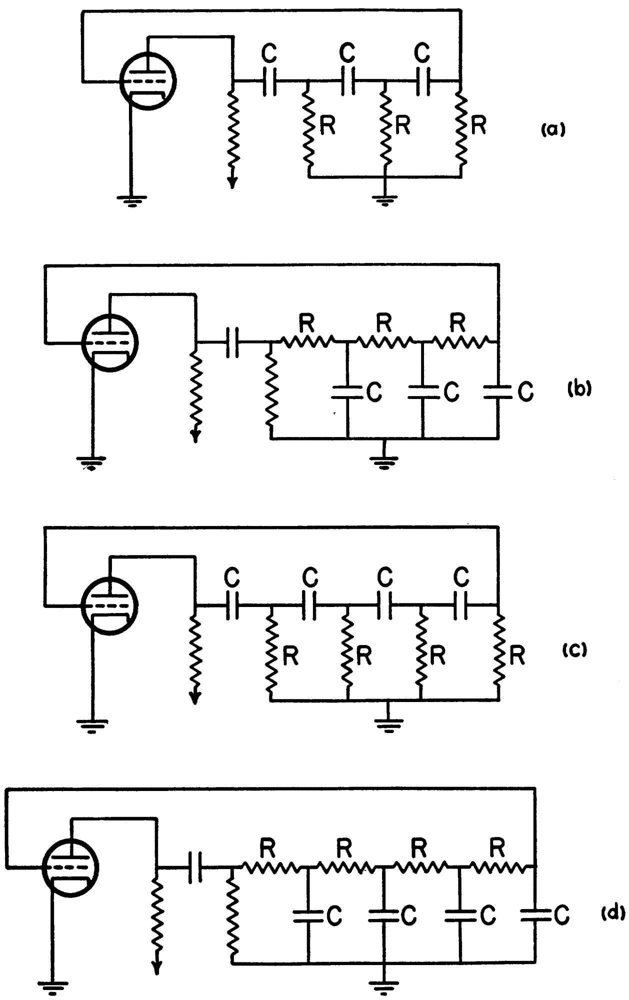



Fig. 1—Basic 3- and 4-mesh phase-shift oscillator circuits.

and a phase-shifting network. It is well known that such an amplifier produces a phase shift of 180 degrees in the frequency range where all shunting capacitances may be neglected. In order to satisfy (4), the phase-shifting network that follows the amplifier tube must produce another 180 degrees of phase shift. As will be easily seen, all networks shown in Fig. 1 are capable of producing this required phase shift.

The voltage amplification $|A|$ which the tube must posess depends upon the type of phase-shifting network used. For instance, in Fig. 1(a), there will be a 180-degree phase shift in the network when $X=\sqrt{6}R$, where $X=1/2\pi fC$, and $C$ and $R$ are as shown in Fig. 1(a). This means that the required phase shift takes place at a frequency at which only 1/29 of the output voltage appears at the grid of the tube. If the tube possesses an amplification equal to or greater than 29, oscillation will start. The second and higher harmonics of the fundamental occur at frequencies at which the phase shift is not 180 degrees but is nearly zero. This means that the harmonics of the fundamental occur at frequencies at which the gain through the phase-shifting network is high (approaches unity) and the feedback becomes negative. The entire arrangement thus becomes equivalent to a negative-feedback amplifier with a feedback ratio of unity. This prevents excessive amplification of harmonics and tends to produce pure sine-wave output.

Fig. 1(b) shows another type of phase-shifting network. In this case, the required phase shift occurs when $X=R/\sqrt{6}$ or when the shunting capacitances have a reactance which is smaller than the resistances $R$. At the second and higher harmonics, the capacitances have even smaller reactances in comparison to $R$ and the amplification of harmonics is small in comparison with the amplification of the fundamental.

In Fig. 1(c) an oscillator circuit is shown which is similar in principle of operation to that of Fig. 1(a) with the exception that a 4-mesh phase-shifting network is used instead of a 3-mesh network. This change merely requires a different amplification in the amplifier to cause oscillation. Fig. 1(d) is similar to Fig. 1(b) in the same way. In general, 3 or more meshes may be used. Two meshes cannot provide enough phase shift without requiring infinite amplification for oscillation.

## Theoretical Analysis of a Typical Circuit

### (*a*) *Frequency of Oscillation and the Necessary Amplification*

To illustrate how the frequency of oscillation and the required amplification may be determined the circuits of Fig. 2 are used. Fig. 2(a) shows an oscillator of the type illustrated in Fig. 1(a). Fig. 2(b) shows an equivalent circuit of Fig. 2(a), with the connection between the output and the input omitted. $R_p$ and $\mu$ are the plate resistance of the tube and its amplification factor, respectively. In order to find the frequency of oscillation it is necessary to compute the total amplification of the circuit $A\beta$ and find the frequency at which this equals unity. Let the grid-to-ground voltage be equal to $e_0$ and the output voltage at the end of the phase-shifting network be equal to $e$. Then $e/e_0=A\beta$, and this ratio must equal unity to cause the amplifier to oscillate.

The equivalent diagram may be simplified further by the use of Thevenin's theorem. The part of the circuit to the left of points $x-x$ in Fig. 2(b) may be replaced by an equivalent voltage in series with the impedance looking to the left of $x-x$ with the source of voltage $e_0$ short-circuited. This results in the equivalent diagram shown in Fig. 2(c) which may be analyzed in the conventional manner. Referring to Fig. 2(c) the following set of equations may be written:

$$\begin{aligned} i_1(R_1 + R - jX) - i_2R + i_30 &= E \\ -i_1R + i_2(2R - jX) - i_3R &= 0 \\ i_10 - i_2R - i_3(2R - jX) &= 0. \end{aligned} \tag{5}$$

Solving these simultaneously for $i_3$, one finds that

$$i_3 = \frac{R^2}{R^3 - 5RX^2 + R_1(3R^2 - X^2) + j(X^3 - 6R^2X - 4RR_1X)} E. \tag{6}$$

The voltage appearing across the output of the phase-shifting network is $e = i_3R$. Hence,

$$\frac{e}{E} = \frac{1}{1 - 5\left(\frac{X}{R}\right)^2 + \frac{R_1}{R}\left[3 - \left(\frac{X}{R}\right)^2\right] + j\left[\left(\frac{X}{R}\right)^3 - 6\left(\frac{X}{R}\right) - 4\left(\frac{R_1}{R}\right)\left(\frac{X}{R}\right)\right]}. \tag{7}$$

Equation (7) gives the ratio of the voltage across the output of the phase-shifting network to the voltage at its input. In order to satisfy the conditions for oscillation outlined before, this ratio must be a real negative number. Hence,

$$\left(\frac{X}{R}\right)^3 - 6\left(\frac{X}{R}\right) - 4\left(\frac{R_1}{R}\right)\left(\frac{X}{R}\right) = 0 \tag{8}$$

$$\frac{X}{R} = \sqrt{6 + 4\frac{R_1}{R}}. \tag{9}$$

Or, since

$$X = \frac{1}{2\pi fC},$$

$$f = \frac{1}{2\pi RC\sqrt{6 + 4\frac{R_1}{R}}}. \tag{10}$$

Equation (10) determines the frequency at which oscillation will take place in terms of the constants of the phase-shifting network. If $R \gg R_1$, then

$$f = \frac{1}{2\sqrt{6}\,\pi RC} \tag{11}$$

which was the formula used in the qualitative discussion.

The gain necessary for oscillation may be determined from (7). Since the imaginary part of this equation is zero at the frequency of oscillation,

$$\frac{e}{E} = \frac{1}{1 - 5\left(\frac{X}{R}\right)^2 + \frac{R_1}{R}\left[3 - \left(\frac{X}{R}\right)^2\right]}. \tag{12}$$

But $E = Ae_0$. In order to produce oscillations, $e_0$ must equal $e$. Therefore,

$$-\frac{1}{A} = \frac{1}{1 - 5\left(\frac{X}{R}\right)^2 + \left(\frac{R_1}{R}\right)\left[3 - \left(\frac{X}{R}\right)^2\right]}. \tag{13}$$

Combining (10) and (13),

$$A = 29 + 23\frac{R_1}{R} + 4\left(\frac{R_1}{R}\right)^2. \tag{14}$$

If $R \gg R_1$, (14) becomes

$$A = 29. \tag{15}$$

From (11) it is seen that the frequency of oscillation depends upon the first power of $R$ and $C$, and not the square root as would be the case with an inductance-capacitance resonant circuit. This is of particular interest in variable-frequency oscillators where it is often desirable to produce a 10:1 change in frequency in

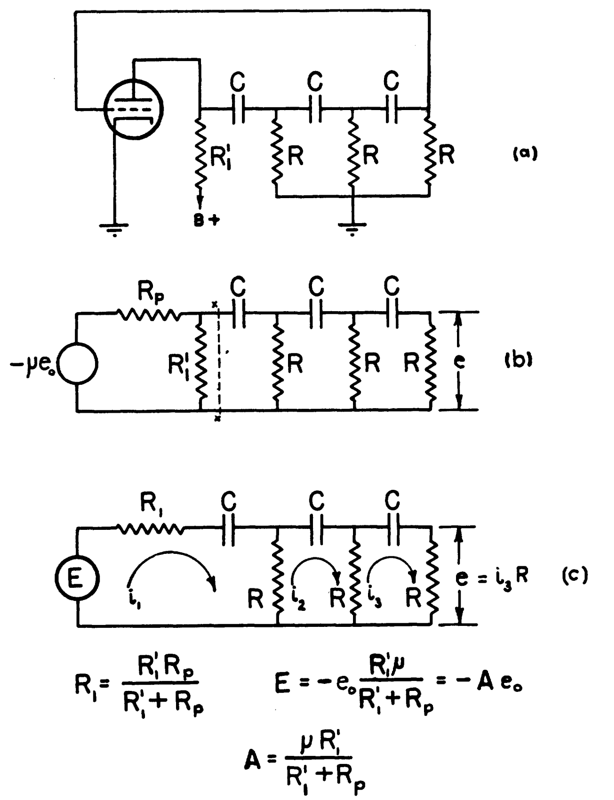



Fig. 2—Equivalent circuits for analysis of phase-shift oscillator.

one rotation of the tuning dial. Equation (15) states that if $R \gg R_1$, the gain required for oscillation is independent of frequency. Because of this fact, the output voltage from the oscillator remains constant regardless of the frequency of oscillation.

### (b) *Frequency Stability*

The frequency of oscillation is determined by the constants of the phase-shifting network and the resistance $R_1$ (see Fig. 2). In fact, all of the circuit equations show that the frequency of oscillation varies inversely with the product of $R$ and $C$. Both of these quantities may change with temperature so that frequency stability depends upon the temperature coefficients of the resistors chosen for $R$ and the condensers chosen for $C$. In the case of variable condensers the change of capacitance with temperature is low (about 15 parts per million per degree centigrade) so that its effect is negligible. Fixed condensers if used may be of recent types which have very nearly zero change of capacitance with temperature. The resistors $R$ should also be chosen to have a low temperature coefficient of resistance.

Variations in supply voltage produce an inevitable change in the plate resistance of the tube which introduces a change in the resistance $R_1$. The resultant change in the frequency of oscillation may be computed as follows: Let $df$ be a small change in frequency $f$ that takes place due to $dR_1$, a small change in resistance that is a result of a variation in the plate resistance of the tube. Frequency stability may be defined on a percentage basis:

$$\frac{\text{change in frequency of oscillation}}{\text{frequency of oscillation}} = k_1 \frac{\text{change in resistance } R_1}{R_1}$$

or,

$$\frac{df}{f} = k_1 \frac{dR_1}{R_1} \tag{16}$$

where $k_1$ is a coefficient of stability. If $k_1 = 0$, a small change in $R_1$ would produce no change in frequency. As an example, the frequency stability of the simple oscillator is computed below. Equation (10) states that the frequency of oscillation of the circuit shown in Fig. 2 is

$$f = \frac{1}{2\pi RC\sqrt{6 + 4\dfrac{R_1}{R}}}\cdot \tag{10}$$

Differentiating (10) with respect to $R_1$,

$$\frac{df}{dR_1} = \frac{1}{2\pi RC} \times \frac{-2}{R\left(6 + 4\dfrac{R_1}{R}\right)^{3/2}} \tag{17}$$

or,

$$\frac{df}{f} = -\frac{R_1}{2R_1 + 3R} \times \frac{dR_1}{R_1}\cdot \tag{18}$$

By comparing (18) and (16),

$$k_1 = -\frac{R_1}{2R_1 + 3R}\cdot \tag{19}$$

Referring to the notation shown in Fig. 2,

$$R_1 = \frac{R_1' R_p}{R_1' + R_p}\cdot \tag{20}$$

Expressing frequency stability in terms of $R_1'$ and $R_p$ by making this change in variables, (18) becomes

$$\frac{df}{f} = -\frac{R_1'^2}{\left[2R_1' + 3R\left(1 + \dfrac{R_1'}{R_p}\right)\right][R_1' + R_p]} \times \frac{dR_p}{R_p}\cdot \tag{21}$$

A stability coefficient $k_2$ may be defined as before, so that

$$\frac{df}{f} = k_2 \frac{dR_p}{R_p} \tag{22}$$

$$k_2 = -\frac{R_1'^2}{\left[2R_1' + 3R\left(1 + \dfrac{R_1'}{R_p}\right)\right][R_1' + R_p]}\cdot \tag{23}$$

For example, a typical design might lead one to the following set of constants:

$R_1' =$ 10,000 ohms $\quad R = 1{,}000{,}000$ ohms
$R_p = 1{,}000{,}000$ ohms $\quad f = 1000$ cycles per second

Substituting these values in (23), one finds that $k_2 = -3.3 \times 10^{-5}$. Thus, a 10 per cent change in the plate resistance of the amplifier tube would result in a frequency change of $3.3 \times 10^{-4}$ per cent, or 0.003 cycle per second. The frequency stability of such an oscillator is better at low than at high frequencies as can easily be verified by comparing (23) and (10).

Experiments with an oscillator similar to the one shown in Fig. 2 have demonstrated that 100 per cent increase in the supply voltage will not produce a noticeable change in the frequency as indicated by a Lissajous figure on a cathode-ray oscilloscope.

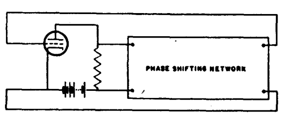
## Variable-Frequency Oscillators

The basic circuits shown in Fig. 1 are equally well suited for constant or for variable-frequency oscillators. If one wishes to construct a single-frequency oscillator the resistances can be approximately equal and the capacitances can also be approximately equal. The frequency may then be adjusted by small variation of one of the condensers or resistors in the phase-shifting network. Actually, the frequency of oscillation may be varied by a considerable amount in this manner.

If a variable-frequency oscillator is desired, the resistors $R$ or the condensers $C$ in Fig. 1 may be ganged together. In a laboratory oscillator it is often desirable to calibrate the tuning dial from 1 to 10 and change the frequency range by turning a decade switch. This can best be done by using variable condensers whose capacitance can be changed by 10:1 and arranging a switch which changes the resistances by decade steps.

The frequency of the oscillator circuits shown in Fig. 1 can also be changed by large amounts without ganging all the condensers or all the resistors. For instance, in Fig. 1(a), the first two condensers $C$ have a common point and, therefore, they may be replaced by a standard 2-gang variable condenser with the rotor insulated from ground. The third condenser $C$ may have a value which is an average of the maximum and the minimum values of the ganged condenser. An oscillator of this type is easier to construct than one in which three condensers or resistors must be ganged. But two things happen as a result of this procedure. Obviously enough, for a certain change of capacitance in the variable condenser the tuning range is smaller if only two instead of three condensers are varied. The gain required for oscillation also changes in a manner depending upon how much the variable capacitances deviate from the fixed one. These undesirable results often are not too serious and it seems quite practical to use oscillators of this type.

Fig. 3 presents the various kinds of phase-shifting networks that seem to the authors to be most practicable, together with the frequency of oscillation and the necessary amplification. From this figure it will be seen that if all condensers are ganged together, the necessary gain is independent of frequency, whereas, if one of the condensers is fixed, the necessary amplification will change. This means that some sort of automatic volume control must be provided to maintain a constant amplitude and a low distortion.



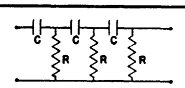
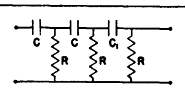
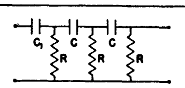
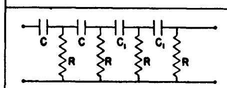
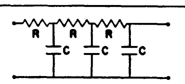
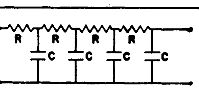

| PHASE SHIFTING NETWORK | FREQUENCY OF OSCILLATION | NECESSARY AMPLIFICATION |
|---|---|---|
|  | $=\frac{1}{2\pi\sqrt{6}RC}$ | 29 |
|  | $=\frac{1}{2\pi RC_1\sqrt{\frac{C}{C_1}\left(5+\frac{C}{C_1}\right)}}$ | $16+10\frac{C_1}{C}+3\frac{C}{C_1}$ |
|  | $=\frac{1}{2\pi RC_1\sqrt{3\frac{C}{C_1}\left(1+\frac{C}{C_1}\right)}}$ | $14+3\left(\frac{C_1}{C}+4\frac{C}{C_1}\right)$ |
|  | $=\frac{1}{2\pi RC_1\sqrt{\frac{3\left(\frac{C}{C_1}\right)^2+7\frac{C}{C_1}}{4+3\frac{C}{C_1}}}}$ | $\frac{9\left(\frac{C}{C_1}\right)^3+33\left(\frac{C}{C_1}\right)^2+55\left(\frac{C}{C_1}\right)+90+84\left(\frac{C_1}{C}\right)}{\left(4+3\frac{C}{C_1}\right)^2}$ |
|  | $=\frac{\sqrt{6}}{2\pi RC}$ | 5 |
|  | $\frac{\sqrt{\frac{10}{7}}}{2\pi RC}$ | 18.4 |

Fig. 3—Design equations for phase-shift oscillators.

For the sake of making the interpretation of some of the results given in Fig. 3 easier, the variation of frequency and the necessary amplification as a function of the tuning capacitances are shown graphically in Figs. 4 and 5. It will be noted that if a large tuning range for a certain capacitance variation is desired, a large change in amplification must be tolerated. On the other hand, if a smaller change in frequency is acceptable the necessary amplification does not change appreciably.

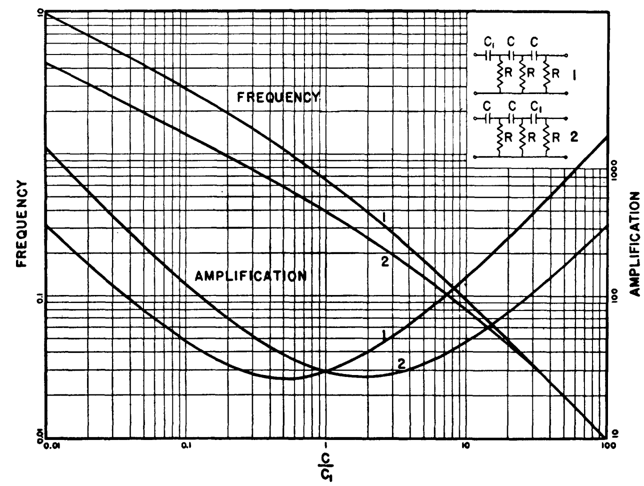



Fig. 4—Frequency of oscillation and required amplification as a function of capacitance ratio for a phase-shift oscillator employing a 3-mesh circuit.

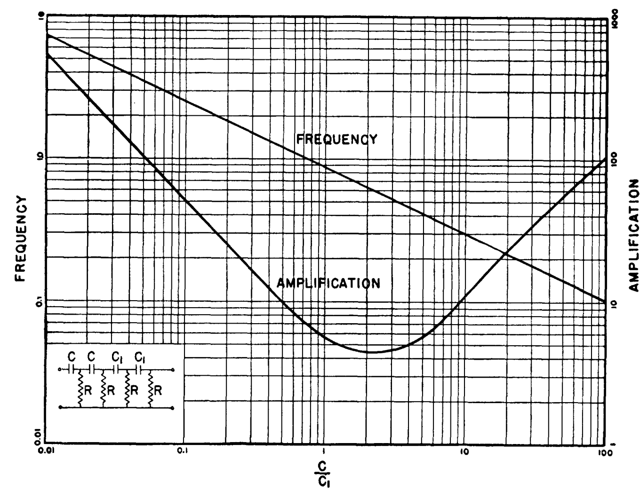



Fig. 5—Frequency of oscillation and required amplification as a function of capacitance ratio for a phase-shift oscillator employing a 4-mesh circuit.

## Typical Designs

Fig. 6 shows a schematic diagram of a variable-frequency oscillator built by the authors intended for use in a communications laboratory. It consists of a pentode amplifier using an 1851 tube so that sufficient gain was developed without an excessively large coupling resistance $R_1'$. The phase-shifting network of the type shown in Fig. 1(a) was chosen because this particular circuit requires smaller $R$ or $C$ than other possible circuits for a given frequency. Physical limitations prevented the use of a variable capacitance much larger than 800 micromicrofarads. In order to produce a frequency of 30 cycles per second one finds that $R=1/\sqrt{6}(2\pi\times 30\times 800\times 10^{-12})=2.7\times 10^6$ ohms. This value of $R$ is almost too high to be inserted in the grid circuit of a tube, and for this reason, circuits of the type shown in Fig. 1(a) are preferable.

As will be seen in Fig. 6, only the first two condensers in the phase-shifting network are variable. This, of course, decreases the tuning range that can be obtained from a single rotation of the condenser dial. As a matter of fact, circuit constants in Fig. 6 were so proportioned that only a 3.3:1 change in frequency

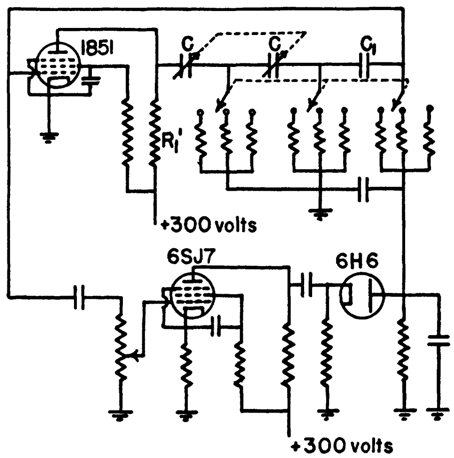



Fig. 6—Circuit of a phase-shift oscillator with automatic volume control.

resulted from a 10:1 change in capacitance. This necessitated two calibrations of the tuning dial, one from 1 to 3.33, and the other from 3.33 to 10. The double calibration of the dial is not necessarily a disadvantage for it effectively doubles the length of the tuning scale for a given frequency band and permits a more accurate calibration. The value of the constant $C_1$ was so chosen that a minimum variation in amplification took place over the tuning range. The amplification of the tube was controlled by means of a delayed automatic volume control which limited the strength of oscillation to 20 volts root-mean-square.

The disadvantages of this simple circuit are as follows: On the highest frequency range the resistance $R$ has a value which is nearly the same as the resistance $R_1'$. This means that the frequency calibration of the tuning dial is different on the highest range from those ranges where $R_1'$ is negligible in comparison with $R$. The fact that $R$ shunts $R_1'$ also means that a higher amplification must be developed by the tube which tends to introduce distortion on other ranges where the necessary gain is not affected by the presence of $R$. Also, frequency stability is not so good as it might be. This is obvious from (23).

These disadvantages may be largely eliminated by adding another tube to the circuit, as shown in Fig. 7. The first tube, a 6AB7-1853 supplies the gain necessary for oscillation. The next tube, a 6AG7, acts as an impedance transformer. The voltage developed between the cathode and ground is almost equal to the grid-to-ground voltage and is of the same phase. As far as frequency of oscillation and the necessary gain is concerned, this tube may be entirely neglected. But the impedance looking back from the phase-shifting network is $[R_k(1/g_m)]/[R_k+(1/g_m)]$ where $g_m$ is the transconductance of the tube, and $R_k$ is the cathode-to-ground resistance. Since $g_k$ for 6AG7 is approximately 5000 micromhos, this impedance is about 200 ohms, whereas, in the circuit shown in Fig. 6, the resistance seen by the phase-shifting network was about 15,000 ohms. Therefore, by using the impedance transformer of this type, *the effect of the tubes on the performance of the circuit becomes negligible.*

If a balanced output from such an oscillator is desired, a resistance equal to $R_k$ may be introduced into the plate circuit of the 6AG7. Thus, the tube serves not only to separate the frequency-determining network from the tube which supplies the amplification but also acts as a phase inverter to provide a balanced output voltage which may be utilized to drive a push-pull output stage directly. An oscillator of this type built in the laboratory was found to be capable of delivering 15 volts root-mean-square from each side to ground with 0.5 per cent second-harmonic and 0.08 per cent third-harmonic distortion. A delayed automatic volume control of the type shown in Fig. 6 was also used in the oscillator shown in Fig. 7. The output voltage remained constant within $\pm 5$ per cent over the frequency range of the oscillator (50 to 40,000 cycles per second).

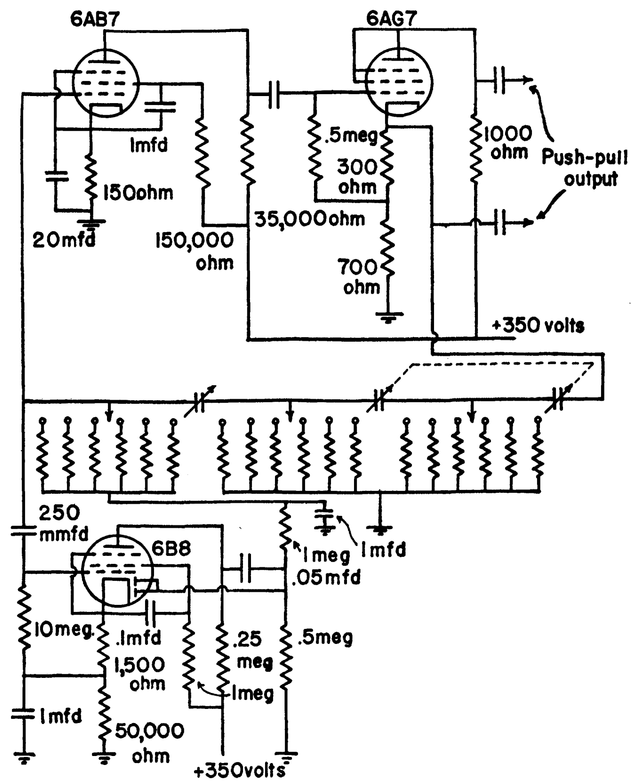



Fig. 7—Circuit of a phase-shift oscillator with automatic volume control and impedance-transformer tube.

## Conclusions

Two things can be said for the oscillators described in this paper. If one desires a constant audio-frequency oscillator, it is probably impossible to devise any cheaper or simpler circuits than those shown in Fig. 1. If one desires a high-quality, variable-frequency oscillator, it will be found that an oscillator similar to the one shown in Fig. 7 is as good as any available on the market or described in the literature. In certain applications it may actually prove to be superior, especially in cases where frequency stability (with reference to supply-voltage fluctuation) is an important factor.

# Fluctuations Induced in Vacuum-Tube Grids at High Frequencies*

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***Summary***—*A theoretical formula for the noise induced in the input circuit of vacuum-tube amplifiers by fluctuations in the electron stream is compared with measured values. The results are found to be in substantial agreement.*

IT IS generally appreciated that a varying electron stream will induce alternating currents in near-by conductors. Therefore, it is to be expected that, in addition to the other well-known fluctuation phenomena found in vacuum tubes at moderately low frequencies,[1] the random variations in space current will induce current fluctuations in the control-grid circuit, giving rise to grid-voltage fluctuations proportional to the total input impedance (tube and circuit) and, for small transit angles, to the frequency. Indeed, Ballantine[2] predicted such an effect in 1928, and it was observed by R.C.A. Communications engineers at Riverhead in the course of a study of the sources of noise in receiving circuits operating in the neighborhood of 10 to 20 megacycles.

Nyquist's[3] well-known formula for the mean-square short-circuit current fluctuations in a passive network exhibiting a shunt conductance $g$ (any susceptance whatever) at the frequency in question is

$$\overline{i^2} = 4kTg\Delta f$$

in which $T$ is the absolute temperature of the network; $k$, Boltzmann's constant; and $\Delta f$, the band width. The results of a theoretical investigation of induced grid-current fluctuations $i_g$ by one of us (D.O.N.) may be expressed in similar form, thus

$$\overline{i_g^2} \approx \frac{20}{3}\left(1 - \frac{\pi}{4}\right) 4kT_k g_g \Delta f = 1.43(4kT_k g_g \Delta f)$$

in which $T_k$ is the cathode temperature and $g_g$ is that portion of the tube input conductance *traceable to electronic loading alone*[4,5] (as distinguished from leakage, dielectric losses, and particularly, feedback effects). The formula refers specifically only to tubes of conventional proportions, operating at frequencies such that transit angles are not greater than a radian or two, and exhibiting at low frequencies the space-charge-reduced, cathode-current shot effect discussed elsewhere.[1]

For comparison with thermal agitation in passive networks at room temperature $T_0$, the formula may be written

$$\overline{i_g^2} = \beta(4kT_0 g_g \Delta f)$$

where

$$\beta = 1.4 \frac{T_k}{T_0}.$$

With $T_0 \approx 300$ degrees Kelvin and $T_k \approx 1000$ degrees Kelvin, a temperature representative of sleeve-type cathodes,

$$\beta \approx 4.8.$$

We have attempted experimental checks of this figure with tubes equivalent to standard tube types 6AB7, 6SK7, 6J5, and 6AC5. Measurements at 30 and at 100 megacycles showed values of $\beta$ ranging from 3.5 to 6.5 and averaging about 5. Attempts to improve experimental accuracy by more careful determination of average cathode temperatures were not very significant. The indicated proportionality of $\beta$ to $T_k$ was confirmed in the range, 900 to 1200 degrees Kelvin.

An important part of the experimental procedure was the use of a bridge-balance scheme devised by one of us (W.R.F.) to eliminate feedback. This scheme made it possible to obtain a true measure of $g_g$ with specially constructed tubes having regular production parts but provided with an additional pair of leads, to cathode and anode, respectively. Lead lengths and interelectrode capacitances in many tubes are such that failure to neutralize feedback results in an order-of-magnitude error in the experimental determination of $\beta$.

* Decimal classification: R132. Original manuscript received by the Institute, January 13, 1941. In order to bring the practical aspects of this subject immediately before those interested in the design of high-frequency receivers, the usual description of analytical and experimental procedures has been omitted. It is the purpose of the authors to supply these details in a subsequent publication.

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[2] S. Ballantine, "Schrot effect in high-frequency circuits," *Jour. Frank. Inst.*, vol. 206, pp. 159–167; August, 1928.

[3] H. Nyquist, "Thermal agitation of electric charge in conductors," *Phys. Rev.*, vol. 32, pp. 110–113; July, 1928.

[4] W. R. Ferris, "Input resistance of vacuum tubes as ultra-high-frequency amplifiers," PROC. I.R.E., vol. 24, pp. 82–105; January, 1936.

[5] D. O. North, "Analysis of the effects of space charge on grid impedance," PROC. I.R.E., vol. 24 pp., 108–136; January, 1936.