Reduction of Power System Magnetic Field by Configuration Twist

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Abstract: The low-frequency magnetic field in the vicinity of helical three-conductor arrangements carrying symmetric three-phase alternating current is studied by analytical numerical and experimental methods. New approximate and simple closed-form expressions are derived for all three components of the three-dimensional field vector as function of current, distance, pitch and conductor spacing. Comparison is made with numerical results. Measurements on an experimental helical arrangement has also been done for verification.

Keywords: Magnetic field, twisted line, helix, power cables, power lines.

1. INTRODUCTION

Twisting of conductor pairs is a well known method used in telephone communications to minimize crosstalk among lines in bundled cables. The effectiveness of the method is based on two phenomena. One is that twisting renders each line immune to the magnetic field coming from the other lines. The other phenomenon is that the field emission from each of the lines is lowered by twisting. This effect of twisting can generally be utilized when the magnetic field is a problem. As an example, twisting has been used in telemetry applications to reduce the stray fields from power-supply leads. The potential of the method for low-field high-voltage power transmission has also been recognized, see e.g. [1] for a futuristic overhead line application. Already practised is twisting of insulated high-voltage three phase power cables, a technique that is directly applicable to low-voltage building wiring. Another potential application is substation busbars.

Despite its importance, the theory of twisted lines has been surprisingly sparsely covered in the literature. Review discloses that an analytical solution for the twisted pair in form of an infinite series containing Bessel functions was given by H. Buchholz as early as 1937, [2], and later expounded on in [3]. This pioneering work was, as it seems, long unknown to the American electromagnetic compatibility community, which instead quote [4] with a similar solution as the main reference, e.g. [5] studying the one-phase two-wire case and [6] the three-phase three-wire case. These papers as well as [7], correcting [5] and referencing [2], deal also with approximations to the rather bulky series-type solution. Except for [8] and [9], which attempt direct ways of approximation, all known papers utilize that the first term of the series is dominant in the practically interesting cases.

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Unfortunately, many of the papers mentioned contain certain errors which makes direct use for field prediction hazardous. The object of this paper, therefore, is to present a revised and extended theory for both two-wire and three-wire twisted lines. It will then be seen that the characteristics of the field have at the same time qualitative differences and similarities which seems to have been overlooked. The analytic theory will be supported by numerical evaluation as well as experiment.

2. THE EXACT THEORY

2.1 Procedure

Exact expressions for the static and quasi-static field from the one-wire, two-wire and three-wire helix will be given. The current is supposed to be filamentary and the configuration infinitely extended in both ends. As will be shown, the two- and three-wire cases can directly and in basically the same way be derived from the one-wire case once the solution of this is known.

2.2 Single wire helix

According to the law of Biot-Savart, the magnetic flux density vector of a helical line source carrying current I is given by the line integral

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{r}' \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3} \tag{1}$$

along the helix, where $\vec{r}$ is the field point and $\vec{r}'$ the source point variable, see Fig. 1. In the figure, a is the radius of the cylinder on which the conductor can be considered to be *wound*, and p is the pitch of the helix. MKSA-units are used so that B is given in Teslas. The constant μ_0 is the permeability of free space $(4\pi \cdot 10^{-7})$. This integral cannot be calculated analytically in a direct manner. However, the integrand can be series expanded in terms whose integrals can be given in the form of Bessel functions. In cylinder co-ordinates r, ϕ and z, see Fig. 1, the radial, azimuthal and axial components B_r, B_ϕ and B_z, respectively, are

$$B_r = \frac{\mu_0 I a}{\pi r^2}(kr)^2 \sum_{n=1}^{\infty} n I_n'(nka) K_n'(nkr) \sin\left[n(\phi - \phi_0 - kz)\right] \quad (2)$$

$$B_\phi = \frac{\mu_0 I}{2\pi r} + \frac{\mu_0 I a}{\pi r^2}(kr) \sum_{n=1}^{\infty} n I_n'(nka) K_n(nkr) \cos\left[n(\phi - \phi_0 - kz)\right]$$

$$B_z = -\frac{\mu_0 I a}{\pi r^2}(kr)^2 \sum_{n=1}^{\infty} n I_n'(nka) K_n(nkr) \cos\left[n(\phi - \phi_0 - kz)\right]$$

with $k = \dfrac{2\pi}{p}$

Here $I_n(z)$ and $K_n(z)$ are the modified Bessel functions of first and second kind of order n, see [10] and $I_n'(z)$ and $K_n'(z)$ their derivatives. Eq. (2) is by H. Buchholz. The screw-type character of the field is apparent since the field is constant on helices where $(\phi - kz)$ is constant.

The problem has recently been revisited in [11]. Eq. (2) holds for $r>a$ and a similar equation applies to the case $r<a$, i.e. inside the cylinder.

Some limiting properties of the solution are worth mentioning. For very large distances the three Bessel-function sums may be shown to go to zero much faster than the inverse -r term of B_ϕ, so that the only field component

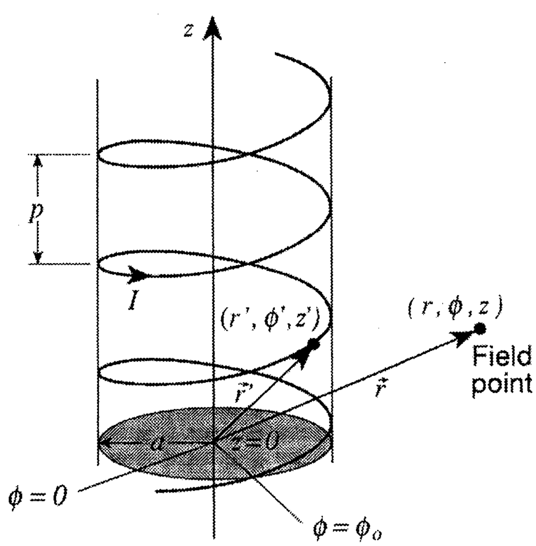

Figure 1 Helical line current

ultimately left will be $B_\phi = \mu_0 I/(2\pi r)$ which is the field of an infinitely long straight conductor carrying current I. This tells that you can not get rid of the field from a single conductor by twisting it. The same field is the boil-down of (2) for any distance when p tends to infinity, so that the helix degenerates into a line.

Another extreme case is when p tends to 0. If I is supposed to tend simultaneously to 0 while I/p is kept constant, the current distribution will approach a purely azimuthal surface current on the cylinder surface. In this case all three field components vanish, which was to be expected from the theory of solenoids where the field is concentrated to the inner of the cylinder.

2.3 Two-wire helix

Without loss of generality, we may set $\phi_0 = 0$ for the conductor carrying current I and $\phi_0 = \pi$ for the conductor of current $-I$. The field from each of the helices is given by (2), and the total field is found by summation. Thus as the even order terms will cancel pairwise while the odd will double, we arrive at

$$B_r = 2\frac{\mu_0 I a}{\pi r^2}\gamma^2 \sum_{n}^{\infty} n I_n'(n\eta) K_n'(n\gamma) \sin\left[n(\phi - kz)\right] \quad (3)$$

$$B_\phi = 2\frac{\mu_0 I a}{\pi r^2}\gamma \sum_{n}^{\infty} n I_n'(n\eta) K_n(n\gamma) \cos\left[n(\phi - kz)\right]$$

$$B_z = -2\frac{\mu_0 I a}{\pi r^2}\gamma^2 \sum_{n} n I_n'(n\eta) K_n(n\gamma) \cos\left[n(\phi - kz)\right]$$

with $\eta = \dfrac{2\pi a}{p} \qquad \gamma = \dfrac{2\pi r}{p}$

where the summations range over n=1,3,5. These are the expressions derived in the pioneering work [2]. For sinusoidal current of angular velocity ω, I is to be replaced by $\hat{I}\sin(\omega t)$, where $\hat{I}$ is the peak value. Up to sign, the expressions will also hold as they stands for the effective values of the B-components with I denoting the effective value of the current.

2.4 Three-wire helix

We label the wires by i=1, 2 and 3 and define their locations ϕ_i and current phase angles α_i by $\phi_i = (i-1)2\pi/3$ and $\alpha_i = (i-1)2\pi/3$. The currents are then $I_i = \hat{I}\sin(\omega t + \alpha_i)$. Term-wise addition of the three fields yields as factors inside the summation signs in (3)

$$\sum_{i=1}^{3} \sin(\omega t + \alpha_i)\sin\left[n(\phi - \phi_i - kz)\right] = \mp\frac{3}{2}\cos(\omega t \pm n\Phi) \quad (4)$$

$$\sum_{i=1}^{3} sin(\omega t+\alpha_i)cos[n(\phi-\phi_i-kz)]=\frac{3}{2}sin(\omega t\pm n\Phi)$$

with $\Phi=\phi-kz$

for n=1,2,4,5... and zero for n=3,6,9..., where the upper sign applies for n=2,5,8... and the lower for n=1,4,7.... Thus

$$B_r=\frac{3}{2}\frac{\mu_0 \hat{I}a}{\pi r^2}\gamma^2\sum_n(\mp n)I_n'(n\eta)K_n'(n\gamma)\,cos(\omega t\pm n\Phi) \qquad (5)$$

$$B_\phi=\frac{3}{2}\frac{\mu_0 \hat{I}a}{\pi r^2}\gamma\sum_n nI_n'(n\eta)K_n(n\gamma)\,sin(\omega t\pm n\Phi)$$

$$B_z=-\frac{3}{2}\frac{\mu_0 \hat{I}a}{\pi r^2}\gamma^2\sum_n nI_n'(n\eta)K_n(n\gamma)\,sin(\omega t\pm n\Phi)$$

with the index and sign convention of above. The effective values of the field components and the total field B can be seen to be given by

$$B_r=\frac{3}{2}\frac{\mu_0 Ia}{\pi r^2}\gamma^2\left[\sum_n\sum_m(\mp n)(\mp m)I_n'(n\eta)I_m'(m\eta)\,K_n'(n\gamma)K_m'(m\gamma)\,cos(\pm n\mp m)\Phi\right]^{1/2} \qquad (6)$$

$$B_\phi=\frac{3}{2}\frac{\mu_0 Ia}{\pi r^2}\gamma\left[\sum_n\sum_m nmI_n'(n\eta)I_m'(m\eta)\,K_n(n\gamma)K_m(m\gamma)\,cos(\pm n\mp m)\Phi\right]^{1/2}$$

$$B_z=\frac{3}{2}\frac{\mu_0 Ia}{\pi r^2}\gamma^2\left[\sum_n\sum_m nmI_n'(n\eta)I_m'(m\eta)\,K_n(n\gamma)K_m(m\gamma)\,cos(\pm n\mp m)\Phi\right]^{1/2}$$

3. APPROXIMATIONS

3.1 The first-term approximation

For certain cases of parameter values for a and p, the first term of the series expansions will be so dominant that it can serve as an approximation for the whole sum for certain values of the variable r. Two limiting cases will be studied in particular. For both cases we assume that the configuration is loosely twisted in the meaning that $a<<p$. This probably will be the case in all feasible power system applications. One of the limiting cases is the untwisted line achieved by letting p tend to infinity seen at a large distance so that $r>>a$ but with $r<<p$. The other limiting case is with the field-point very distant so that $r>>p$. For this case it will show that the effective values of the total field for the twisted and straight cases, B and B_0 can be written as $B\approx FB_0$. B_0 is different for the two- and three-wire cases, but F will be the same. F is called "twist factor".

The two- and three-wire cases will be treated in a parallel way. The procedure followed will include extracting the first term of the Bessel series for each field component. In this way we achieve an intermediate approximation which will not be used as it stands but will be further approximated using approximations to the Bessel factors.

3.2 The two-wire case

The first-term field components given in effective value scale are from (3)

$$B_r=-2\frac{\mu_0 Ia}{\pi r^2}\gamma^2 I_1'(\eta)K_1'(\gamma)\,sin(\phi-kz) \qquad (7)$$

$$B_\phi=2\frac{\mu_0 Ia}{\pi r^2}\gamma I_1'(\eta)K_1(\gamma)\,cos(\phi-kz)$$

$$B_z=2\frac{\mu_0 Ia}{\pi r^2}\gamma^2 I_1'(\eta)K_1(\gamma)\,cos(\phi-kz)$$

An approximate expression for the effective value of the total field can now be given, but is not shown in this presentation. It is observed that the screw-form field structure of the single wire helix is retained.

To find the field for the untwisted case we use the small argument approximations of the pertinent Bessel functions according to

$$I_1'(\eta)\approx\frac{1}{2} \qquad K_1(\gamma)\approx\frac{1}{\gamma} \qquad K_1'(\gamma)\approx-\frac{1}{\gamma^2}$$

Then, for any z

$$B_r\approx B_0\,sin\,\phi \quad B_\phi\approx B_0\,cos\,\phi \quad B_z\approx 0 \quad B\approx B_0 \qquad (8)$$

with $B_0=\frac{\mu_0 Ia}{\pi r^2}$

For the twisted case we use the large argument approximations to the Bessel functions according to

$$K_1(\gamma)\approx-K_1'(\gamma)\approx\sqrt{\frac{2}{\pi\gamma}}e^{-\gamma}$$

Then, using again the approximation for $I_1'(\eta)$ of above, we arrive at

$$B_r\approx FB_0\,sin(\phi-kz) \quad B_z\approx FB_0\,cos(\phi-kz) \quad B_\phi\approx 0 \qquad (9)$$

$$B\approx FB_0$$

with $F=\sqrt{\frac{\pi}{2}}\gamma^{3/2}e^{-\gamma}$

It is noted that the twist-factor applies individually to the component fields as well with z=0 and the ϕ- and z-coordinates interchanged.

3.3 The three-wire case

The approximated effective value field components are from (6)

$$B_r = -\frac{3}{2}\frac{\mu_0 Ia}{\pi r^2}\gamma^2 I_1'(\eta)K_1'(\gamma) \tag{10}$$

$$B_\phi = \frac{3}{2}\frac{\mu_0 Ia}{\pi r^2}\gamma I_1'(\eta)K_1(\gamma)$$

$$B_z = \frac{3}{2}\frac{\mu_0 Ia}{\pi r^2}\gamma^2 I_1'(\eta)K_1(\gamma)$$

Contrary to the two-wire case, this field does not possess a screw-form structure, but is constant for constant r independent of ϕ and z.

For the untwisted case we have

$$B_r \approx B_\phi \approx B_0/\sqrt{2} \qquad B_z \approx 0 \qquad B \approx B_0 \tag{11}$$

with $B_0 = \frac{3}{4}\sqrt{2}\frac{\mu_0 Ia}{\pi r^2}$

corresponding to (8) for the two-wire case.

For the twisted case we have

$$B_r \approx B_z \approx FB_0/\sqrt{2} \quad B_\phi \approx 0 \quad B \approx FB_0 \tag{12}$$

with $F = \sqrt{\frac{\pi}{2}}\gamma^{3/2}e^{-\gamma}$

corresponding to (9) for the two-wire case. We see that the twist-factors are identical, which was prospected. Also in this case we have that F applies individually to the component fields with ϕ and z interchanged. We further note that the total three-phase field is higher by a factor of $3\sqrt{2}/4 \approx 1.06$.

As an illustration of what field reduction can be brought about by twist of a triangular configuration, we see that for distance to pitch ratios r/p equal to 1 and 2 F is equal to 0.037 and 0.00020, respectively. This shows the extreme fastness in field decay with distance. Different, and much less dramatic twist-factor values are presented in [1], where the total field decay seems to be of inverse cubic form rather than exponential. This discrepancy could maybe be explained by that a too short line may have been used.

4. VERIFICATIONS

4.1 Scope

The object here is to verify the correctness of the Bessel-function expansion formulas and to demonstrate the precision of the approximate method. A laboratory experiment was also performed to show the correspondence between the theoretical solution and reality.

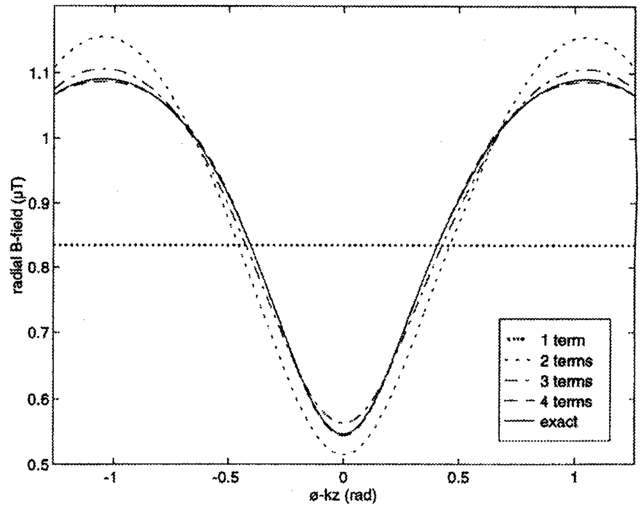

Figure 2 Analytical prediction of radial component of magnetic field

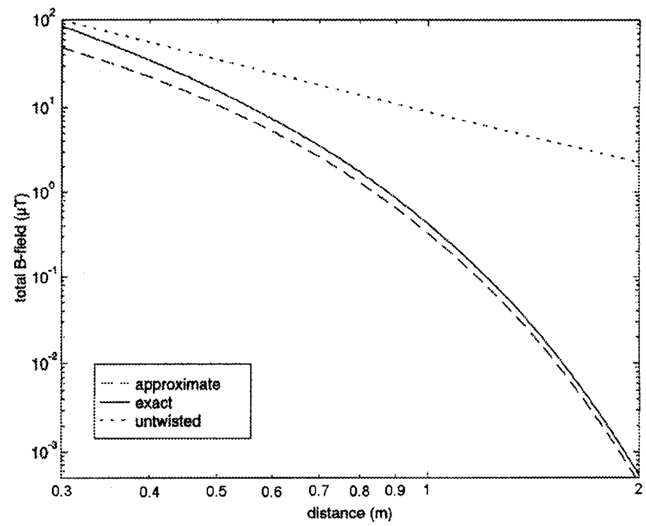

Figure 3 Approximate, exact and untwisted total magnetic field

4.2 Analytical solution vs numerical

In order to guarantee the correctness of the analytical solution presented in the paper, comparison has been made with a numerical solution based directly on (1). The agreement was found to be excellent in all cases tested, but is not demonstrated in this paper.

4.3 Precision of the approximation

Fig. 2 shows, for a certain three-phase case, the series expansions of B_r of (6) based on 1, 2, 3 and 4 terms. Here a=0.1 m, p=1 m, r =0.2 m, and I=1 A. The comparison is made for varying (ϕ-kz) covering all possible field points on the cylinder. Here, the 1-term case corresponds to the approximation according to (10). It is seen that the variation with (ϕ-kz) is quite significant in this near field region, but that the convergence is fast when terms are added. Anyhow, the first-term approximation seems to be a good estimate of the averaged field. Corresponding diagrams for greater distances would show that the variations are gradually wiped out. The same will apply to the other two components.

Fig. 3 shows the approximation according to (12) applied to the configuration above with I=200 A alongside of the exact solution for varying r. It appears that the approximation works well even when γ is not very large. As a rule of thumb, the error will be smaller than 10% when $\gamma>10$ as long as $\eta<0.5$. Also shown is the untwisted case. The difference between the approximate and the untwisted cases is just the twist-factor F.

4.4 Theory vs experiment

A rig consisting of three plastic coated 50 mm² stranded copper wires helically wound on a 11 m long plastic pipe of 0.2 m diameter was constructed. The wires were electrically connected at one end and fed by a balanced 200 A, 50 Hz sine current in the other end, see Fig. 4. The experiment was performed for two pitches: 0.5 m and 1.0 m. The three components were measured at various positions in an axial plane by use of a one-coil magnetic-field meter.

The probe of the meter was mounted on a wooden block designed to keep the center of the coil in an axial plane for all three measuring positions. For each distance, the field at six points spaced 0.1 m was measured and the average value was used for comparison with the prediction of the numerical method, see Fig. 5, for the results with the 1.0 m pitch. The agreement is seen to be excellent in a near zone. The deviations for larger distances was found to originate mainly from deviations of the windings from perfect helices by

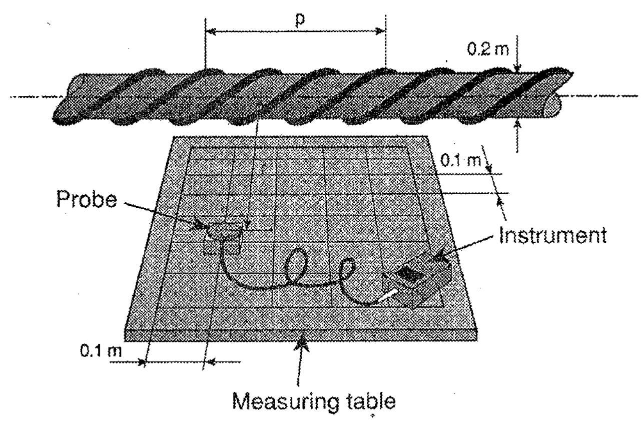

Figure 4 Experiment set up

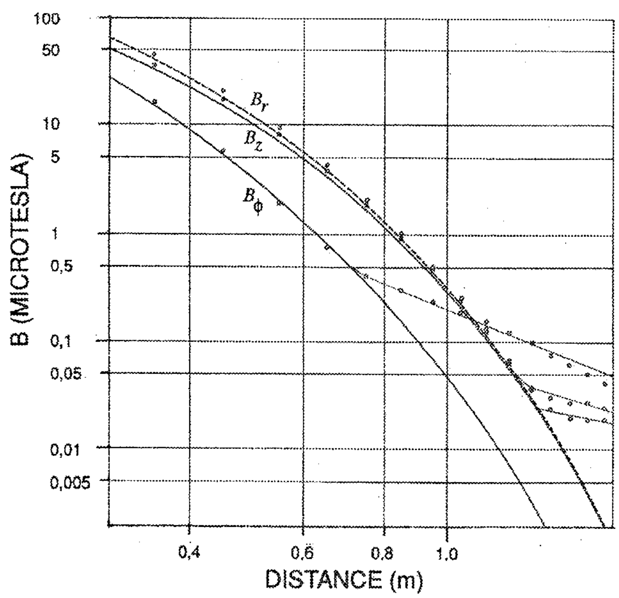

Figure 5 Experimental (designated by circles) and theoretical magnetic field components

experimenting, and slight adjustments of these would have pushed the range of agreement up to even greater distances until the ambient field and maybe also any unsymmetry in the three-phase current would be limiting. In any case, comparing with the calculated field of an untwisted configuration, the F-factor actually achieved for the total field is 0.042 at r=1 m as compared with the theoretical value 0.037. For r above, the F achieved will decrease, even if not as much as theoretically predicted . In effect this decrease will be about inverse cubic with distance, which would indicate a small localized deviation from a perfect helix.

5. DISCUSSION

It is interesting to compare configuration twist with other field reducing options. One such is phase-split, meaning that the current of each of the phases is distributed equally on two subphases placed diametrically on a circle or in a corresponding way, see e.g. [9]. By this, the exponent of r for the far field decay is raised from two to three. By further phase split, in principle any exponent can be achieved. But twisting beats eventually for large distances any such arrangement as the field decay is exponential!

Further, for uninsulated conductors, the phase-split method entails proportional enlargement of the radius with degree of splitting to maintain insulation, which will compromise the reduction of the near field. In this context it is interesting to note that the influence of the radius a on the field magnitude is contained in that of the untwisted reference field B_0, implying that the field is not more than proportional to the radius.

6. CONCLUSIONS

The power-frequency magnetic fields of two-wire one-phase and three-wire three-phase twisted conductor configurations have been studied, in order to assess the potential as a field reducing option in power systems. A review of the literature on electromagnetic compatibility revealed that H. Buchholz presented an analytical solution for the field of the generic infinite current-carrying one wire helix as early as 1937. The form of the solution is an infinite series of products of modified Bessel functions. Departing from this solution, the field of the two-wire and three-wire cases can easily be established by superposition.

Since this series-type solution is rather bulky, approximations are wanted. One approximation used in the literature consists in keeping only the first term of the sum as an approximation to the complete sum. The paper addresses two important limiting cases where such a first-term approximation is valid. One of these cases is when the distance to the line is much smaller than the pitch of the helix but at the same time much greater than the radius of the helix, in which case the field of the configuration degenerates into that of a straight line. The other and less trivial case is when the pitch is much smaller than the distance but much greater than the radius. In this case a so called "twist-factor" of the form

$$F = \sqrt{\frac{\pi}{2}}\gamma^{3/2}e^{-\gamma} \qquad \gamma = \frac{2\pi r}{p}$$

can be derived. F is the field reduction achieved by twisting a straight line. Here r is the distance from the line and p the pitch of the helix.

The paper, which revises some earlier research, shows that F applies to the field of the three-wire case as well as to that of the two-wire case. One important difference, however, is that while the two-wire field retains its screw-type variation for the radial and axial field components, the corresponding variations will vanish in the three-wire case. The azimuthal field component vanishes in both cases.

The analysis is supported by numerical simulations departing directly from the Biot-Savart law and by laboratory experiments on a rig of the three-phase case.

7. REFERENCES

[1] Zaffanella, L., "Magnetic Field Management for Overhead Transmission Lines; A Primer", EPRI TR-103328, Dec. 1994

[2] Buchholz, H., "Elektrische Strömungsfelder mit Schraubenstruktur", Elektrische Nachrichtentechnik, pp. 264-280, 1937

[3] Buchholz, H., "Elektrische und Magnetische Potentialfelder", Chapter 6, Springer-Verlag, Berlin/Göttingen/Heidelberg, 1957

[4] Sensiper, S., "Electromagnetic Wave Propagation on Helical Conductors", MIT Research Laboratory of Electronics, Tech. Rep 194, 1951

[5] Moser, J.R., Spencer, R.F., "Predicting the Magnetic Field from a Twisted Pair Cable", IEEE Trans. Electromagn. Compat., Vol. EMC-10, pp. 324-329, Sept. 1968

[6] Haber, F., "The Magnetic Field in the Vicinity of Parallel and Twisted Three-wire Cable Carrying Balanced Three-Phased Current", IEEE Trans. Electromagn. Compat., Vol. EMC-16, No. 2, pp. 76-82, May 1974

[7] Shenfeld, S., "Magnetic Fields of Twisted-Wire Pairs", IEEE Trans. Electromagn. Compat., Vol. EMC-11, No. 4, pp. 164-169, Nov. 1969

[8] Alksne, A.Y., "Magnetic Fields near Twisted Wires", IEEE Trans. Space Electr. Telem., Vol. SET 10, pp. 154-158, Dec. 1964

[9] Pettersson, P., "Principles in Power System Magnetic Field Reduction", Stockholm Power Tech, Paper SPT HV 12-01-0428, pp. 358-363, June 1995

[10] Abramowitz, M., and Stegun, I.A., "Handbook of Mathematical Functions", Washington D.C., U.S. Government Printing Office, 1964

[11] Hagel, R., Gong, L., Unbehauen, R., "On the Magnetic Field of an Infinitely Long Helical Line Current", IEEE Trans. Magn., Vol. 30, No. 1, pp. 80-84, Jan. 1994

BIOGRAPHY

Per Pettersson (M'90) was born in Sweden in 1942. He received the M.Sc. degree in Electrical Engineering from the Chalmers Institute of Technology, Gothenburg, Sweden in 1967, and 1973 the Technical Licentiate degree in Mathematical Statistics from the same institute. He received the Ph.D. degree in Electrical Plant Engineering in 1995 from the Royal Institute of Technology, Stockholm, Sweden. Dr. Pettersson is presently with Vattenfall Utveckling AB as a Senior Research Engineer.

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