Optimum Air Gap for Various Magnetic Materials in Cores of Coils Subject to Superposed Direct Current

V. E. LEGG
MEMBER AIEE

Synopsis: A method is developed for calculating optimum air gap and core size for choke coils whose windings must carry direct current in addition to alternating current. It is employed with data on typical magnetic materials to give curves from which design dimensions can be obtained readily to fulfill specified requirements for inductance, resistance, and d-c burden.

THE PROBLEM of proper proportioning of air-gap and metallic-path length of the magnetic core of choke coils and transformers frequently arises in the design of coils whose windings must carry direct current along with the desired alternating current. This problem has been approached in the past along lines laid down by Hanna.[1] This involved calculations based on known characteristics of the magnetic materials and taking into account the effects of air gaps of various lengths inserted in the magnetic circuit. Taking an estimatedly correct core volume, V, the optimum value of applied field then was computed, from which the number of turns in the coil was obtained. The resulting coil resistance then was calculated to see if it was satisfactory. If not, a new core volume was assumed, and the calculations were repeated until the desired winding resistance was attained.

It has seemed that a more direct approach to optimum dimensions could be devised, which would give an immediate answer as to the suitability of any given core material for a specified inductance, resistance, and d-c burden. An analysis of the problem has been made, and curves have been plotted for several important magnetic materials from which design dimensions can be readily obtained.

General Problem: L, R, and I Specified

The general method is based on the known fact that prerequisites of inductance L, resistance R, and d-c burden I, can be met by proper expansion or contraction of the size of a core, leaving the ratios of dimensions fixed. Thus, for example, all dimensions can be expressed as fixed multiples of the metallic core length. The essential equations then can be set up and solved for minimum core length. This gives relationships between the (μ, B) curve, (μ_r, B) curve, and the specified L, R, and I of the coil, which must be met for the optimum conditions. The necessary metallic path length, air-gap percentage, and number of turns in the coil follow directly.

Considerable study has been given by others to determine the ratios of dimensions in various types of coils to apportion space properly to windings and magnetic materials. The present analysis assumes that technically or commercially approved dimension ratios are known for whatever coil and core configuration may be considered and confines itself to determination of core and air-gap relationships. If the metallic-path length is l, the average length of a turn of wire in the winding will be ul, the area available for copper winding will be vl^2, and the cross-sectional area of the magnetic core will be wl^2, where u, v, and w are known constants. The length of the air gap is similarly αl, where the factor α is to be adjusted to make l as small as possible. Air gaps will be initially assumed so short as to avoid fringing, so that the flux density in the air is as great as that in the metal. Necessary adjustments to take account of fringing with longer air gaps will be described.

With these assumptions the specified resistance R, inductance L, and d-c burden I enter into the equations as follows.

The resistance of the copper winding will be proportional to the total length of wire ($N \times$ average turn length, ul) and inversely proportional to the cross-sectional area per turn, vl^2/N, or

$$R=\frac{\rho Nul}{vl^2/N}=\frac{u\rho N^2}{vl} \text{ ohm} \qquad (1)$$

where N is the number of turns, l is in centimeters, ρ is the resistivity in ohm-centimeters, and the copper area constant v takes into account the packing factor and insulation of the winding.

The inductance, as measured with alternating current of small amplitude, depends upon the permeability of the metal and the relative length of air gap. The permeability to be used is the a-c permeability at the value of superposed field set up by the direct current in the winding. For simplicity only the reversible permeability μ_r will be considered. The results for values of alternating current differing from zero will approximate quite closely those obtained for zero current, namely, μ_r. This follows from the fact that the a-c permeability changes less with changing current in the presence of superposed field than without The inductance is derived from the basic equation

$$L=\frac{N\Phi_m}{i_m}\times 10^{-8} \text{ henry}$$

where Φ_m is the peak flux in maxwells corresponding to the a-c peak i_m, in amperes. The flux can be written as the ratio of magnetomotive force, $0.4\pi Ni_m$, to the reluctance, $l/wl^2\mu_r+\alpha l/wl^2$, whence

$$L=\frac{4\pi N^2wl}{l\div\mu_r+\alpha}\times 10^{-9}=4\pi N^2\mu_e wl\times 10^{-9} \qquad (2)$$

where μ_e is the effective permeability, that is, the permeability of a uniform core material occupying the same space as the

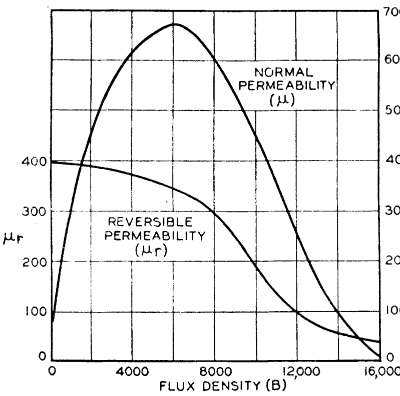

Figure 1. Normal and reversible permeability of four-per-cent silicon iron

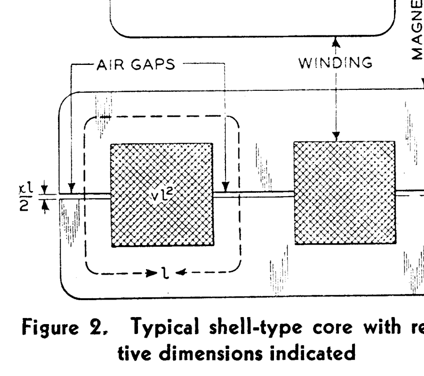

Figure 2. Typical shell-type core with relative dimensions indicated

Paper 45-149, recommended by the AIEE committee on communication for publication in AIEE TRANSACTIONS. Manuscript submitted February 23, 1945; made available for printing July 27, 1945.

V. E. LEGG is in the apparatus development department, Bell Telephone Laboratories, Inc., New York, N. Y.

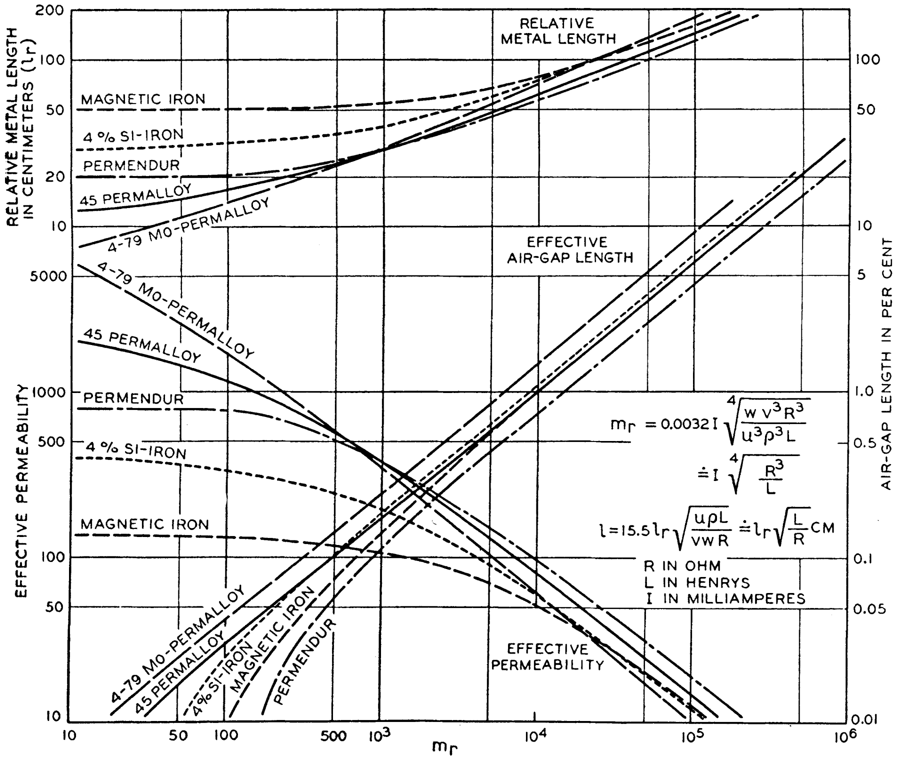

Figure 3. Optimum air gap, relative metal-path length, and effective permeability for magnetic cores subject to superposed d-c field

core in question and yielding the same inductance. Substituting N^2 from equation 1 gives

$$1/\mu_r+\alpha=\frac{4\pi vwRl^2}{u\rho L}\times 10^{-9}=kl^2 \qquad (3)$$

where

$$k=(4\pi vwR/u\rho L)\times 10^{-9}$$

The magnetomotive force due to direct current in the winding is distributed between the magnetic core and the air gap so as to maintain the steady flux density B in the core by means of field strength $H=B/\mu$, and in the air gap by field strength $H_a=B$, neglecting fringing in the gap. The magnetomotive force is thus

$$lB/\mu+\alpha lB=0.4\pi NI$$

Substituting N from equation 1 gives

$$\frac{1}{\mu}+\alpha=\frac{0.4\pi I}{B}\sqrt{\frac{Rv}{u\rho l}}=\frac{m}{B\sqrt[4]{kl^2}} \qquad (4)$$

where

$$m=4\pi I\sqrt[4]{(4\pi R^3v^3w\div[u^3\rho^3L])}\times 10^{-13}$$

Equations 3 and 4 contain essentially three unknowns—B, α, and l (μ and μ_r are functions of B). The final equation needed to determine these unknowns and to satisfy the condition for smallest possible coil is $dl/dB=0$, all other dimensions having been assumed proportional to powers of l. Eliminating α between equations 3 and 4 gives

$$B\sqrt{kl^2}(1/\mu-1/\mu_r+kl^2)=m \qquad (5)$$

Differentiating with respect to B and setting $dl/dB=0$ gives

$$kl^2=\frac{1}{\mu_r}-\frac{1}{\mu}+B\left[\frac{1}{\mu^2}\frac{d\mu}{dB}-\frac{1}{\mu_r^2}\frac{d\mu_r}{dB}\right]=\frac{1}{\mu_r}-\frac{1}{\mu}+B\Delta \qquad (6)$$

where

$$\Delta=\left[\frac{1}{\mu^2}\frac{d\mu}{dB}-\frac{1}{\mu_r^2}\frac{d\mu_r}{dB}\right]$$

Substituting this value of kl^2 back into equation 5 gives

$$B^2\Delta\sqrt[4]{B\Delta+1/\mu_r-1/\mu}=m \qquad (7)$$

This equation gives the optimum value of B for the desired value of m, but it also implies selection of corresponding values of μ, μ_r, $d\mu/dB$, and $d\mu_r/dB$, to satisfy the equation. These functions must be determined from (μ, B) and (μ_r, B) curves such as shown in Figure 1 for four per cent silicon iron. On account of the involved nature of the data, equation 7 is handled most readily in reverse direction, by inserting known values of B, μ, μ_r, and so forth, from the magnetic data and solving for the constant m. Such calculations from the curves of Figure 1 are given in Table I.

The value of the air-gap ratio is obtained from equations 3 and 6 as

$$\alpha=B\Delta-1/\mu \qquad (8)$$

The metallic-path length from equation 6 is

$$l=\sqrt{(B\Delta+1/\mu_r-1/\mu)/k} \qquad (9)$$

A function which reveals the influence of the air gap is the effective permeability, μ_e. Thus, from equations 2 and 8

$$\mu_e=\frac{1}{1/\mu_r+\alpha}=\frac{1}{B\Delta+1/\mu_r-1/\mu} \qquad (10)$$

The necessary number of turns in the winding follows from equations 1 and 9.

It is now feasible to plot pertinent data of air-gap and metal length for any desired magnetic material against values of m. From such curves essential dimensions of any coil can be read off immediately when m has been computed. For practical purposes it is convenient to use values for m corresponding to typically proportioned cores. The parameters in the foregoing equations are

$$m=4\pi I\times 10^{-6}\sqrt[4]{4\pi R^3v^3w/(10u^3\rho^3L)} \text{ and}$$

$$k=4\pi vwR\times 10^{-9}\div(u\rho L)$$

where I is in milliamperes, R in ohms, ρ in ohm-centimeters, L in henrys, and l is in centimeters. The constants u, v, and w will have various values, depending on the shape of the core, whether annular, shell type, or otherwise. Under the assumption that shell-type cores as illustrated in Figure 2 are most common for use with air gaps, typical cores have been measured for values of the dimension ratios. Thus $u=1.8$, $v=0.02$, and $w=0.04$, approximately. Inserting these values in the above equations gives

$$\left.\begin{array}{l} m=4.17\times 10^{-3}IR\div\sqrt[4]{RL} \\ \text{and} \\ k=3\times 10^{-6}R/L \end{array}\right\} \qquad (11)$$

For cores having such typical dimension ratios, it is convenient to express results in terms of

$$m_r=IR\div\sqrt[4]{RL}=m\div(4.17\times 10^{-3})$$

Table I. Data and Computations for Four-Per-Cent Silicon Iron

B(gauss)	2,000	4,000	6,000	8,000	10,000	12,000	15,000
μ	4,520	6,150	6,740	5,950	4,550	2,600	430
$d\mu/dB$	1.03	0.45	0	−0.56	−0.85	−1.00	−0.42
μ_r	386	369	343	294	188	95	20
$d\mu_r/dB$	−0.007	−0.010	−0.017	−0.034	−0.068	−0.031	−0.008
$\Delta\times 10^9$	98	85	144	377	1,889	3,292	17,720
$kl^2\times 10^3$	2.56	2.89	3.63	6.25	24.0	49.5	313.5
m	0.0882	0.315	1.27	6.77	74.2	2,230	29,900
m_r	21.15	75.6	305	1,625	17,800	535,000	7,170,000
α (per cent)		0.018	0.072	0.285	1.87	3.90	26.3
l_r	29.2	31.0	34.7	45.6	89.5	128.5	
μ_e	386	346	276	160	41.6	20.2	3.2
LI^2/V	0.478	5.09	59.4	740	11,780	3,120,000	
$(N/l)\sqrt{V/L}$	451	479	536	704	1,383	1,985	
$NI/l\times 10^{-3}$	0.312	1.082	4.13	19.18	150.0	3,760	

Values of m_r have been computed as given in Table I for four per cent silicon iron. They have been used as abscissas in Figure 3, for plotting optimum air gap and effective permeability of four per cent silicon iron. Computations similar to those in Table I based on published magnetic data[2] have been made for magnetic iron, Permendur, 45 Permalloy, and 4–79 molybdenum-Permalloy, and the results plotted in Figure 3.

An additional family of curves is shown in Figure 3, labeled relative metal length, l_r. The necessary metal length from equation 9 involves k, for which an expression is given in equations 11 for typical core dimension ratios. If the ratio R/L is taken as unity, the typical value of k is 3×10^{-6}. With this value "relative metal-path lengths" have been computed from equation 9. The actual core length in any given case then would be $l_r\sqrt{L/R}$.

If other values of u, v, and w are to be used than the assumed typical values the curves of Figure 3 apply if the abscissas are calculated from the equation

$$m_r=(4/4.17)\pi I\times10^{-3}\sqrt[4]{4\pi R^3v^3w/(10u^3\rho^3L)} \tag{12}$$

Similarly the correct metal length l will be obtained from equation 9 in terms of the relative length l_r given in the curves by the equation

$$l=l_r\sqrt{3{,}000u\rho L/4\pi vwR} \tag{13}$$

Special Case I. L, I, and l (or V) Specified

The problem thus far has assumed complete freedom in the choice of dimensions. Frequently, however, the problem may begin with a certain size of core and inquire after the optimum air-gap length to provide a specified inductance in the presence of a given amount of direct current. In this case it will be necessary to adjust the coil resistance along with the air-gap length to satisfy the optimum conditions. The problem becomes that of ascertaining the values of l_r and m which correspond to the optimum conditions as defined by the above equations and which yield the specified inductance and metallic-path length for the specified current.

The metallic-path length l is known in this case, but l_r and R in equation 13 are not known. Substituting the value of R from equation 13 in equation 12 gives

$$\frac{26.6m_r^2}{l_r^3}=\frac{LI^2}{wl^3}=\frac{LI^2}{V} \tag{14}$$

where V is the volume of the core. This resembles the fundamental equation derived by Hanna,[1] but it has the advantage that m_r and l_r can be computed for any given flux density and material by the methods here described. If the first member of equation 14 is computed using associated values of m_r and l_r which already satisfy the optimum conditions, it yields the necessary parameter to insure that the corresponding l_r and α are optimum under the specified conditions of L, I, and l.

With optimum values of l_r and α, the required number of turns in the winding from equations 1 and 13 becomes

$$N=l_r\sqrt{\frac{3{,}000L}{4\pi lw}}=ll_r\sqrt{\frac{3{,}000L}{4\pi V}} \tag{15}$$

The resistance in the winding then follows from equation 1.

The data used for Figure 3 have been analyzed further to find the values of LI^2/V which correspond to the optimum values of m_r, l_r, and α. The results have been plotted in Figure 4, showing values of

$$l_r\sqrt{3{,}000/4\pi}=(N/l)\sqrt{V/L}$$

and α against LI^2/V.

Case II. L, N, (or R), and l (or V) Specified

If inductance is specified, and it is desired to know the percentage air gap required to permit the use of the largest possible direct current in the winding of a coil which is already constructed, the curves in Figure 4 can again be used. All of the factors required to compute $N\div l\sqrt{V/L}$ now are specified. It remains only to locate the indicated point on the $N\div l\sqrt{V/L}$ curve for the material of which the core is constructed, and read the corresponding value of LI^2/V. From the latter value can be calculated the maximum current which can be employed without understepping the specified inductance.

The necessary air-gap length can be read off directly from Figure 4.

Case III. N (or R), I, and l (or V) Specified

If the d-c burden is specified, and the coil is already constructed (that is, l and N or R are fixed) it may be desired to adjust the air-gap length to yield the maximum inductance. Proceeding as in case I, the inductance is solved for in equation 13, and substituted in equation 12, which, together with equation 1, gives as the necessary relation

$$79.6m_r/\sqrt{l_r}=NI/l \tag{16}$$

Proceeding as in case I, the first member of equation 16 is computed for each flux density, using associated values of m_r and l_r. This insures that the specified conditions of N, I, and l are satisfied when

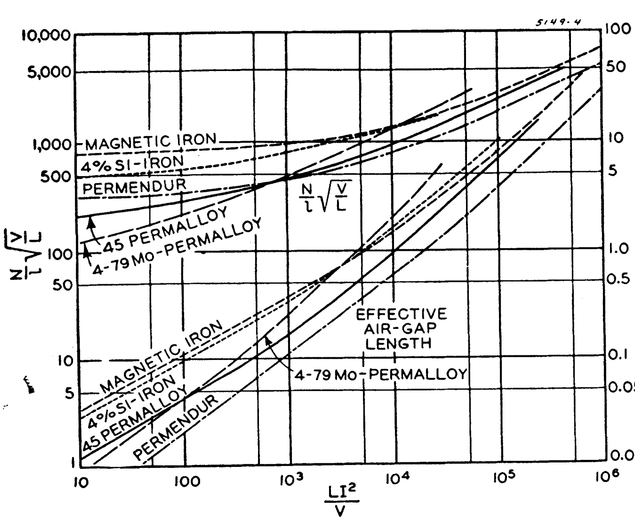

Figure 4. Optimum air gap and number of turns for a coil having specified inductance, core size, and d-c burden

L in henrys
I in centimeters
l in milliamperes

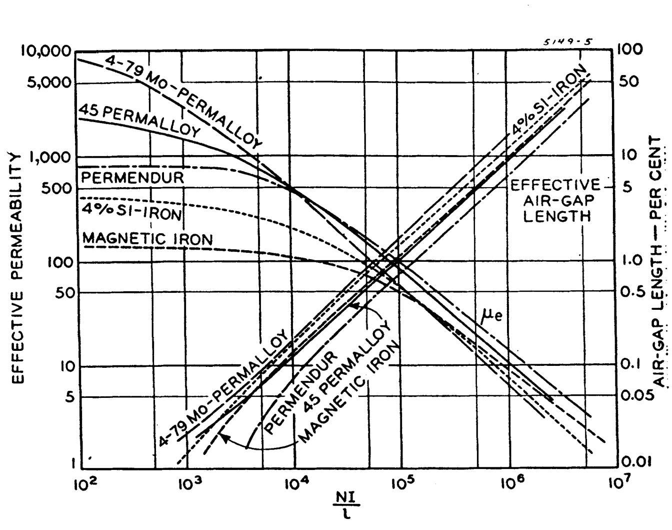

Figure 5. Optimum air gap and permeability for a coil having specified inductance, core size, and d-c burden

L in henrys
I in centimeters
l in milliamperes

the optimum conditions of l_r and α are employed. Curves in Figure 5 give the effective permeability and air gap plotted against NI/l. The inductance attained with the values of μ_e and α so chosen will be the maximum possible. The inductance in henrys is computed from equation 2.

Case IV. N (or R), L, and 1 (or V) Specified

If the inductance is specified and the coil is already constructed, the curves of Figure 5 can be read backwards to find the allowable direct current and the necessary air gap.

Case V. N (or R), 1 (or V), and α Specified

If a finished coil is at hand, the question may arise as to the direct current which should be used to employ the core and coil most efficiently. Curves on Figure 5 may be used in this case to locate the value of NI/l which corresponds to the specified value of α. With this value of NI/l, optimum current is computed. Similarly the corresponding effective permeability is read from the curves, from which the optimum inductance is found.

The Air Gap

The foregoing analysis has been concerned with determining the optimum air-gap length to satisfy certain conditions. When the practical question arises as to the number of millimeters spacing to insert, still further analysis is required, on account of magnetic fringing at the gap. In general a longer physical gap must be inserted into the magnetic circuit than indicated by the optimum percentage. The correction is less for very short gap lengths, or if the total amount of gap is divided into two or more short sections with interspersed core material. The correct gap length can be found empirically by building the coil as designed and then adjusting the air gap to give the calculated inductance with the specified direct current in the windings. An approach to the calculation of leakage around an air gap has been given by Partridge.[3]

Conclusion

Consideration has been given to the general problem of adjusting the size of magnetic core, air-gap length, and number of turns in the winding to fulfill specifications as to inductance, resistance, and d-c carrying capacity. Mathematical means have been worked out for fulfilling specified conditions with a minimum quantity of core materials whose permeability and reversible permeability curves are given. Calculations have been made for several typical materials, to illustrate the method of utilizing this type of analysis.

References

1. Design of Reactances and Transformers Which Carry Direct Current, C. R. Hanna. AIEE Journal, volume 46, February 1927, pages 128–31.

2. Survey of Magnetic Materials and Applications in the Telephone System, V. E. Legg. *Bell System Technical Journal*, volume 18, 1939, page 438.

3. The Inductance of Iron Cored Coils Having an Air Gap, G. F. Partridge. *Philosophical Magazine* series 7, volume 22, 1936, page 665.

Eddy-Current Resistance of Multilayer Coils

T. H. LONG
MEMBER AIEE

Synopsis: Most of the earlier studies of eddy-current resistance have been made by those interested primarily in the effect of compromise transpositions on the losses of rotating equipment. Their results have included formulas for the eddy-current resistance for perfect transpositions, but these have gotten into handbooks in difficult form or have been ignored entirely. Such formulas are presented here in more usable form, with more information on their limitations, and further formulas are derived for minimum losses for a number of different cases. Included is a set of curves showing eddy-current resistance as a function of the number of layers and conductor thickness. Formulas also are included that take into account the differing lengths of conductor in successive layers and that approximate the end losses in a spiral coil on account of radial flux.

The customary approach to the problem of eddy-current-loss calculations for multilayer coils has been to assume a coil side surrounded on three sides by a medium of infinite permeability and no loss and the use of rectangular conductors. There has been one exception to this that is discussed later.

The principal aim of most of these investigations has been to arrive at a loss factor that might be applied to the d-c resistance of slot windings of various types to find the effective or a-c resistance. This has included evaluating the loss factor for the individual layers, which is all that needs to be applied if the layers are in series or if a perfect transposition system is used, and a different loss factor that includes the circulating current caused by compromise transpositions in certain practical windings.

Formulas so derived for slot windings have been applied with reasonable accuracy but some awkwardness to such ends as calculating the loss in transformer coils. It is the purpose of the present work to derive a formula that can be modified readily to account for the difference in layer lengths, and that will lead to simple and general approximate formulas that can be used for a very rapid survey of design possibilities in many cases.

Paper 45-137, recommended by the AIEE committee on basic sciences for publication in AIEE Transactions. Manuscript submitted April 17, 1944; made available for printing May 17, 1945.

T. H. Long is senior research engineer with C. G. Conn, Ltd., Elkhart, Ind.

A paper by Butterworth[7] considers an inductance without the usual partial iron circuit and wound with round conductors. It is shown that the ratio of reactance to resistance is a maximum for a coil so proportioned that the ratio of diameter to length is from 2.5 for low frequencies to 3.2 for high frequencies, the distinction between low and high frequencies being made according to the degree to which skin effect is evident in an isolated conductor. At high frequencies the reactance-to-resistance ratio is improved by a spaced winding. Formulas are derived for losses in such coils and shown to correspond to measured results. It also is shown that for the coils in question the losses are almost entirely due to the radial component of flux rather than the axial component. As these results are limited in their application to single-circuit systems (also end-to-end coupling) of rather limited proportions, they are not considered further here.

Returning to the more usual analysis of eddy-current problems, it has been customary to assume further that:

1. The conductors could be represented by rectangular bars of the type shown in Figure 1A, so disposed that two of their flat surfaces are parallel to the general direction of the leakage flux.

2. That component of leakage flux parallel to the other two flat surfaces could be neglected.

3. That the defect in the length l of net copper (along leakage) compared to the total length of the leakage path l_1 could be accounted for by a fictitious permeability $\mu = l/l_1$ and is constant throughout the coil.

4. That the temperature and resistance of the conductors do not vary through the winding.