The Economical Design of Smoothing Filters

By F. S. Dellenbaugh, Jr., and R. S. Quimby*

The articles by Dr. Dellenbaugh and Mr. Quimby in the February and March issues of QST have emphasized the importance of the first choke of the high-voltage rectifier-filter system with respect to both regulation and smoothing, and have clarified as well as simplified the design of the front end of this generally little-considered item of amateur transmitting equipment. In this, the third article of the series, the smoothing action of the filter system is hauled out and given particular treatment. In line with the trend of the times, it is shown how better smoothing for less money can be realized by intelligent coördination of the numerous factors involved. Absolutely practical, this article deserves better than casual reading. — EDITOR.

THE ripple to be expected at the terminals of a smoothing filter employed with r.a.c. power supply may be calculated with a very fair degree of accuracy. While this article is based upon mathematical analysis, the results are presented in tabular and graphical form. Reduction of fundamental design formulas to extremely simple expressions allows the general characteristics of filter circuits to be discussed without trial and error methods, and leads to proper design for maximum smoothing effect with minimum material.

One of the first things is to define what is meant by ripple. Van Der Bijl considers the terminal ripple as the total change in voltage from maximum to minimum. Other writers have considered the ripple as a superimposed wave on the average or d.c. value of voltage, and speak of ripple as the amplitude of this superimposed wave. In the present case a similar consideration is used, but the values of ripple are expressed as the r.m.s. or effective values of the ripple voltage. This is more closely in accordance with general engineering practice. It is warranted since the final ripple is practically a sine wave with any smoothing satisfactory in use. Under these conditions the amplitude of the ripple will be 1.41 times the effective value used in this paper, and the total variation will be twice the amplitude. Therefore conversion from one to the other is simple, the only confusion being to find out which definition any particular author is using.

A warning against undue saturation in iron-cored chokes is always in order. Unfortunate confusion exists between ratings, actual performance and opinion. The measurement of inductance under actual conditions is difficult without a specially equipped laboratory. All of the inductance values mentioned herein are actual values tested by special bridge or other methods under conditions very closely duplicating those found in operation. For good smoothing *any approach to resonance must be avoided.* This holds for both 60 and 120 cycles. With a balanced single-phase full-wave 60-cycle rectifier, 120 cycles is the lowest frequency to be expected in the output. However, a 60-cycle resonance in the filter may produce oscillations which will unbalance the tubes and cause various disturbances varying from too great a ripple to violent oscillation. This is too complicated to be treated in this article and deserves detailed treatment separately. The general warning is that if the r.a.c. power supply appears erratic, look for resonance in the first section and cure it by increasing the size of the first choke or the first condenser.

Fig. 1 shows the two filter elements separated, the arrows indicate the circuits that become resonant. In the first section simple series resonance is found. In the second section resonance around the circuit (circuital resonance) must be considered. The same formulas hold for both, except that in the case of Fig. 1b the value of the two condensers in series must be used. The formulas are as follows:

$$f = \frac{1}{2\pi\sqrt{LC}} \text{ or, } 1 - \omega^2 LC = 0$$

where f = frequency in cycles per second, L = inductance in henrys, C = capacity in farads, $\pi = 3.1416$ and $\omega = 2\pi f$.

For 60 cycles $\omega^2 = 0.142 \times 10^6$
" 120 " $\omega^2 = 0.570 \times 10^6$
C_1 & C_2 in series $= C_1C_2/(C_1+C_2)$

TABLE I

INDUCTANCE REQUIRED TO RESONATE UNDER GIVEN CONDITIONS

		60 Cycles	*120 Cycles*
Fig. 1a	$C = 2\mu$fd.	3.55	0.88 henrys
Fig. 1b	$C_1 = 2\mu$fd. $C_2 = 2\mu$fd.	7.0	1.8
Fig. 1b	$C_1 = 2\mu$fd. $C_2 = 4\mu$fd.	5.3	1.3
Fig. 1b	$C_1 = 2\mu$fd. $C_2 = 6\mu$fd.	4.7	1.2

Thus, with values of chokes in common use there is little danger of approaching resonance in the second filter section, but there may be very grave danger of getting into trouble with the first filter section. A generally unappreciated difficulty lies in the fact that the inductance is not

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constant over the variations of current during each cycle, but has a cyclic value itself due to changes in iron permeability with changing magnetic flux. If any part of the cyclic value falls into the resonant class, instability results, and os-

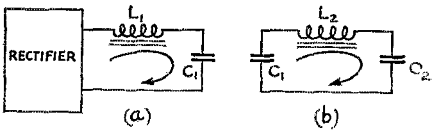


FIG. 1 — RESONANT CIRCUITS IN SMOOTHING FILTER

(a) First section, series resonance. (b) Second and succeeding sections, circuital resonance.

cillographic studies indicate that the cyclic inductance may reach a point as low as 50% of the average value determined by bridge or meter measurements.

The next important item is to define what is meant by "good smoothing." With receiving sets a ripple of 0.01% of plate voltage is inaudible, 0.1% is excellent commercial practice, while 1.0% is bearable, although considerable hum is heard when no signal is being received. As the ripple modulation in the transmitted wave would provide the same relative audibility, these figures probably are approximate measures of satisfaction in transmission as well. Therefore for 'phone transmission we would like to get down to about 1-volt ripple per 1000 volts on the plate, and for telegraphy to 10 volts ripple per 1000 on the plate would probably be satisfactory. These values appear to check roughly with expected smoothing of characteristic circuits as described by various amateurs.

TEST CIRCUIT

The schematic layout of the test circuit used is shown in Fig. 2. The rectifier was the standard '66 type, with the first choke of the swinging variety,[1] having an inductance of 12 henrys under test conditions. The balance of the filter had numerous chokes and condensers arranged so that various sizes and various numbers of sections could be obtained rapidly by test clip connections. The output was fed into a resistance and the a.c. drop across the resistance was measured by a vacuum-tube voltmeter similar to many that have been described in these pages. The method of determining ripple was by deflection and comparison. After the vacuum tube voltmeter measurement was made on the filter circuit, the switch shown in Fig. 2 was thrown over and the required 60-cycle a.c. to give the same deflection was measured by meter and drop wire. This method admittedly is open to some criticism from the standpoint of accuracy and frequency segregation, but serves admirably for a rapid relative method of comparison. It thus meets the present requirements and eliminates errors in calibration of a more elaborate voltmeter. The transformer voltage and load current were maintained the same throughout the tests. The terminal voltage was substantially the same, the only variation being due to changes in choke resistance. Only one set of test voltages and current were used, since the variables are already quite complicated, and there is nothing in the results, either theoretical or practical, that would be changed materially by different power conditions.

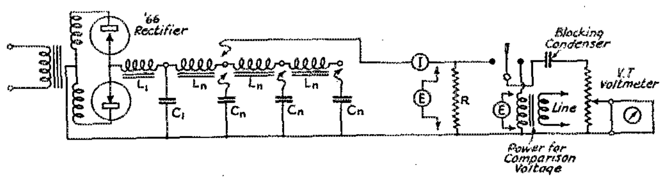


FIG. 2 — GENERAL TEST SET-UP

Adjustable filter circuit and vacuum-tube voltmeter.

TABLE II

FIXED TEST CONDITIONS

Transformer voltage, 2100 volts total (1050 volts per side, r. m. s.).
Terminal voltage, 800 to 860 volts d.c. (depending upon choke resistance).
Load current, 300 milliamperes d.c.
First choke (L), 12 henrys.
Balance of filter circuit adjusted as given in each test.
Load resistance, 2700 to 2900 ohms.

FILTER BEHAVIOR

The voltage delivered by the rectifier to the filter is almost exactly a "folded" sine wave, provided that the first choke is greater than the critical value. The filter elements then smooth out this wave, attenuating the variations to any desired degree, and finally deliver a slightly fluctuating voltage to the load. This is shown graphically in Fig. 3. According to the definitions above, the output is assumed to be a uniform d.c. voltage with a superimposed sine-wave ripple, the measured values being the r.m.s. value of this ripple. The original sine wave from the transformer consists practically of the fundamental frequency only, harmonics usually being absent. When this wave is "folded" by the rectifier, however, it becomes unsymmetrical, and contains a great many harmonics. An analysis shows that 43% of the original branch transformer voltage appears as fundamental in the "folded" sine wave applied to the filter. Since this type of filter has much greater attenuation for higher frequencies, it is this fundamental component which con-

[1] Cf. McLaughlin and Lamb, "What Is This Thing Called Decibel?" *QST*, August, 1931. — EDITOR.

tributes about 99% of the final ripple, and the problem is thus simplified by being able to neglect all other harmonics.

The filter action in smoothing ripple may be considered from three points of view:

1. The "telephone" method of considering its properties of selective attenuation at different frequencies.
2. The energy-storage method, considering the storage capacity of the different elements.
3. The impedance method, considering the over-all impedance of the load plus the filter and the resulting current variations for any impressed voltage wave.

Naturally all three methods give the same result if properly carried out. The first is chiefly useful for considering the pass band and is a little difficult to interpret if the terminal loads do not match the characteristic impedance of the filter. The second gives a very good physical conception of what is happening and is the simplest to use for indicating general characteristics, but more difficult for predetermining ripple. The third is the most complete and direct for smoothing filter computation, but introduces mathematics which somewhat obscure the operating significance until simplified by successive approximations.

Consider the *energy storage conditions*. Inductances store energy in the form of a magnetic field and their energy is thus associated with current. Condensers store energy in the form of electrostatic field, or charge, and thus their energy is associated with voltage. The actual energy is given by the following expressions:

Inductance: Energy $= \frac{LI^2}{2}$ watt-seconds (joules).

Capacity: Energy $= \frac{CV^2}{2}$ watt-seconds (joules).

Resistance: Energy dissipation $= I^2R$ watts or, to match storage form, $= I^2R \times t$ watt-seconds.

Where

L = inductance in henrys, C = capacity in farads, V = potential in volts, I = current in amperes, t = time in seconds.

Inductance or capacity therefore can be considered much in the light of a storage battery, usually rated in ampere-hours; but since the battery voltage is substantially constant, this is also watt-hours. (This is the same unit by which bills are paid to the electric illuminating companies, and so should not be unfamiliar to most of us.) To get an idea of the size and energy involved, consider a 20-henry choke. Suppose it is operating at 1000 volts and a current of 316 ma.; these values are convenient since the square of 316 ma. is 0.1 squared amperes, and the square of 1000 volts is 10^6 which cancels the 10^{-6} in the conversion from farads to μfd. This choke will thus store

$$\frac{LI^2}{2} = 20 \times 0.1 \times 0.5 = 1.0 \text{ watt second.}$$

This is about the size of choke usually used in the second part of the filter circuit. It will weigh about 20 pounds and, if the energy could be properly applied, it would light a 2-watt lamp for a half second. This does not seem like very much energy and offhand one would not expect such small energy storage to do much smoothing. The cause of its utility for this purpose lies in the very short time between cycles. With full-wave 60-cycle rectification the choke is called upon to fill in a valley of only 1/120 second. Then the watts delivered, if assumed uniform, would be $1.0 \times 120 = 120$. This sounds more like something useful. The watts dissipated in the resistance load will be: volts × amperes = 316 watts. So the energy furnished by the choke is a very appreciable part

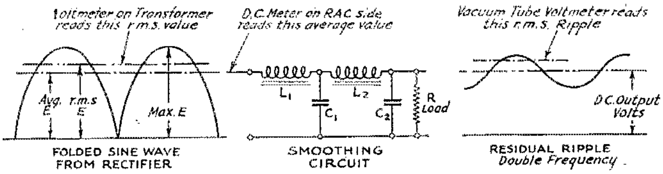


FIG. 3—GRAPHICAL REPRESENTATION OF FOLDED SINE WAVE SMOOTHED BY FILTER CIRCUIT

Full-wave single-phase operation with 2 Type '66 tubes r. m. s. volts = 0.707 max. volts; ave. volts = 0.636 max. volts; r. m. s. volts = 1.11 ave. volts.

of the total energy required. If we associate with the choke a condenser of 2 μfd., this will also store 1 watt-second at 1000 volts; and with such a condenser at each end of the choke there results a total energy storage of 360 watts, just about the same as the energy dissipated.

Generalizing from this, we may say that, roughly, *when the energy stored in the filter elements is equal to the energy dissipated in the load we have fairly good smoothing.*

It usually works out, in cases like this, that if the energy stored in the two kinds of storage elements is equal, then the total material required will be economically utilized. To see what happens we will equate the two storage expressions

$$\frac{LI^2}{2} = \frac{CV^2}{2} \text{ and } V^2 = I^2R^2.$$

We also know that $I = \frac{V}{R}$, so $V^2 = I^2R^2$.

Substituting this above;

$$\frac{LI^2}{2} = \frac{CI^2R^2}{2} \text{ or } L = CR^2.$$

This gives the relation between L and C as

$$R = \sqrt{\frac{L}{C}}.$$

It is interesting that this is the same relation arrived at by the telephone company method of calculation for giving minimum reflections in the pass band, and is what is called the characteristic impedance of the filter.

A filter section made up in this way, with no first choke, gave the following results:

Load voltage, d.c.	860	
Load current, ma.	300	
Load resistance, ohms	2870	
Ripple, volts	16.	
" % of d.c. output	1.9	
Inductance	15	henrys
Storage	0.7	watt-seconds
Condenser, 2 μfd. each end, total	4.0	μfd.
Storage	1.5	watt-seconds
Total storage	2.3	watt-seconds
Total dissipation per cycle	2.15	watt-seconds

Ratio of resistance to $\sqrt{\frac{L}{C}}$, 1.5

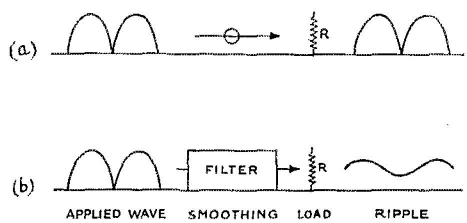


FIG. 4—(A) FOLDED SINE WAVE APPLIED TO RESISTANCE LOAD

with no smoothing gives ripple which is duplicate of impressed wave. (b) Folded sine wave through filter gives reduced ripple; almost entirely fundamental, of double frequency. Ratio of ripple (a) to ripple (b) is attenuation ratio of filter. Attenuation in decibels is

$$20 \log_{10} \frac{Ripple\ (a)}{Ripple\ (b)}$$

It is rather complicated to calculate the actual smoothing to be expected by this method, but the principles involved are well illustrated. Three definite factors result, which frequently are overlooked:

1. Smoothing by inductance results from the value of stored energy, depending upon the inductance and square of the current. This in turn determines the size of the choke, just as the watts output determines the size of a transformer. Thus chokes of 20 henrys at 316 ma., 2 henrys at 1 ampere, and 0.02 henrys at 10 amperes will each store the same amount of energy, will each be of the same approximate size and weight, and will each contribute about the same amount of smoothing to the output.
2. Smoothing by capacity depends upon the stored energy resulting from the capacity in μfd. and the square of the voltage. Thus a 2 μfd. condenser at 1000 volts and a 4000 μfd. condenser at 22.5 volts will store the same energy and contribute about the same amount to smoothing. The so-called dry electrolytic condensers of the latter type are in general about the same in size, weight and cost, as the 1000-volt 2-μfd. condenser.
3. The distribution of energy storage between the inductance and capacity should be approximately adjusted so that the square root of the ratio of henrys to farads is equal to the load resistance. Very close adjustment is not necessary, and it may be that the combination giving the least stored energy for given smoothing will not be the cheapest combination; so, fortunately, a good deal of leeway is allowable.

PREDICTION OF RIPPLE

The third viewpoint mentioned above, that of considering the filter as an added impedance, gives the best results for calculation. The ripple with and without the filter is compared, and the reduction of ripple is considered as the attenuation of the filter. In Fig. 4a is shown a folded sine wave applied directly to a resistance load. The ripple across the resistance obviously will be the same as the output of the filter. The filter, of any type, is now introduced as in Fig. 4b and reduces the ripple. Thus we can introduce a factor called the "overall impedance" of filter and load, designated by Z_r. This is the impedance which will determine the alternating current in the load when any alternating voltage is impressed upon the front end of the filter. As the load is practically a pure resistance, the voltage ripple will be proportional to the current times the resistance. The improvement due to the filter will then be the ratio of the load resistance to the overall impedance. After a number of approximations, the formula reduces to the following simple forms:

For a first choke followed by a single section filter, as is customary practice, the impedance is

$$Z_r = \frac{R \times \omega^4 \times L_1 L_2 C^2}{4}.$$

R.m.s. volts ripple across load $= \frac{0.43E \times R}{Z_r} = \frac{E \times R \times 0.43 \times 4}{R \times \omega^4 \times L_1 L_2 C^2}$ which simplifies to $\frac{1.72\ E}{\omega^4 L_1 L_2 C^2}$ r.m.s. volts ripple.

For 120 cycles, where $\omega^4 = (754)^4 = 32.5 \times 10^{10}$,

volts ripple $= 5.3 \times \frac{E}{L_1 L_2 C^2}$.

Where: R = load res., ohms; $\omega = 2\pi f$; f = frequency, cycles; L_1 = 1st choke, henrys; L_2 = 2nd choke, henrys; C=total capacity, farads; E= transformer volts per side, r.m.s. value; attenuation ratio $= \frac{R}{Z_r}$ db; attenuation $= 20 \log_{10} \frac{Z_r}{R}$.

In the tests given, L was always 12 henrys, so this will simplify still further to

$$\text{volts ripple} = \frac{0.44E}{L_2C^2}, \; C \text{ being } \mu\text{fds.}$$

It is convenient to express the filter effect in various ways:

1. R.m.s. volts ripple gives the actual ripple in the output.
2. The attenuation ratio of R/Z_r gives the reduction of any impressed voltage at the beginning of the filter in a straight ratio.
3. The % ripple in terms of output voltage is convenient for comparing filters operating on different voltages. This will, of course, be the actual voltage ripple divided by the d.c. output voltage of the r.a.c. system.
4. Twenty times the common logarithm of the ratio $\frac{Z_r}{R}$ gives the attenuation of the filter in d.b., which is convenient in dealing with radio problems where d.b. gain after the filter is to be considered.

The d.b. attenuation, while more difficult to visualize, has some other advantages.[1] In considering the amount of ripple desirable it will be found that the amount of material, and in general its cost, will be a substantially constant amount per d.b. This is useful in considering the economics of a whole system, and filters can be manufactured "by the yard," as it were, depending upon the perfection of desired results. Another simplifying feature is that units such as the d.b. can be added directly. Suppose, for example, that the output of a low power modulator is to be amplified by 50 d.b. and that the allowable ripple in the transmitting tube voltage has been found to be about 10 volts. If the modulator is to operate on 200 volts, how good must be the filtering of the plate supply? Now 200 volts d.c. at the filter terminals would mean about 250 volts per branch on the plate supply transformer and 43% of this will be the fundamental ripple applied to the filter, or 107 volts. Only about 10% of this must be allowed to get to the transmitter. The d.b. for a voltage ratio of 10 is 20. The filter must, therefore, have an attenuation of 20 d.b. more than the following amplification, or 70 d.b. The filter attenuation in these terms is given by:

$$\text{d.b. attenuation} = 20 \log_{10} \frac{Z_r}{R}.$$

Or, simplified for the test conditions, d.b. attenuation $= 20\log_{10} 0.081L_1L_2C^2$, and if $L_1 = 12$ henrys, d.b. attenuation $= 20\log_{10} 0.97L_2C^2$.

The approximate formulas hold for most conditions giving satisfactory filtering, say 20% ripple or better. The requirements are that all

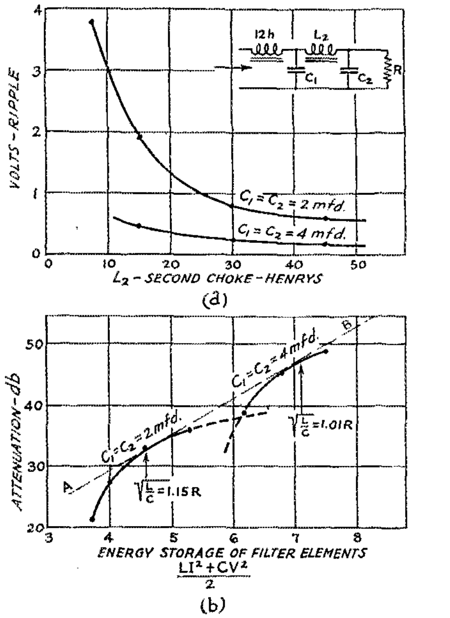


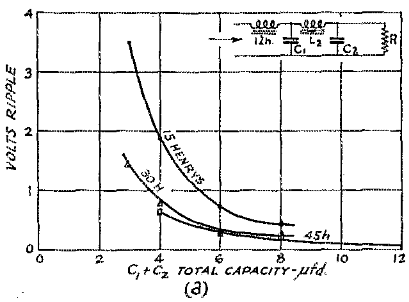
FIG. 5—(A) VOLTAGE RIPPLE FOR VARIOUS VALUES OF INDUCTANCE; DATA FROM TABLE III

(b) Attenuation for various values of energy storage, variable inductance.

circuits must be fairly far from resonance, which are satisfied if ω^2LC is large compared to 1, say at least 4 or 5. Considering the simplest formula with the first choke of definite size, several predictions as to performance are observed.

1. The smoothing is independent of the load resistance. At first glance this seems to be wrong, since practice shows that the ripple decreases for less load, i.e., for greater load resistance. The explanation lies in the fact that what really happens is an increase in the inductance of the chokes due to a smaller direct current. If the proper values of inductance are used in the formula, the proper results will be obtained.
2. The total inductance and capacity produces smoothing, and first and second sections may have different values with the same results. This is only true to a limited extent, partly because of the approximations involved. If the first and second choke are very different in size, better results will be obtained with the smaller choke in the first section. The best smoothing is obtained with the two condensers of equal value, but one may be twice the other in capacity without material difference.

3. The smoothing improves inversely with the square of the total capacity used. This is because the added capacity improves smoothing in both the first and second sections, the attenuation being the product of the values for each section.
4. The smoothing improves inversely as the first power of the second inductance. This is because the first inductance is assumed fixed. If both chokes were increased together, the inductance would appear squared as well as the capacity. As the first choke is determined from other criteria, and as excellent smoothing is obtained without requiring larger values, the best interests of design appear to be served by keeping it at a constant value. However, the formula should *not* be interpreted to mean that capacity is better than inductance for smoothing.



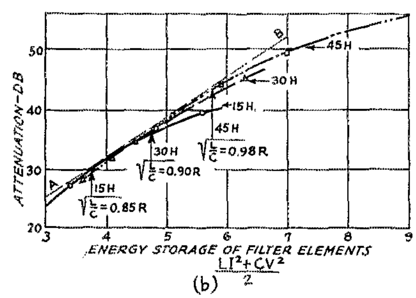


FIG. 6—(A) VOLTAGE RIPPLE FOR VARIOUS VALUES OF CAPACITY, DATA FROM TABLE IV

(b) Attenuation for Various Values of Energy Storage, variable capacity.

5. The voltage ripple varies directly with the impressed volts. This is, of course, to be expected. The attenuation of the ripple is independent of voltage; therefore actual ripple in volts will vary with the initial amount upon which the attenuation operates.

The following tables show comparisons of experimental and calculated results for filters of this type, the above formulas being used. The results are plotted graphically in Fig. 5. The correspondence between test and calculations is unexpectedly good in most cases. It gives the proper magnitude in all cases; which, after all, is the designer's chief aim. Small differences in actual ripple disappear completely when compared with the wide range of decibels used in the radio art.

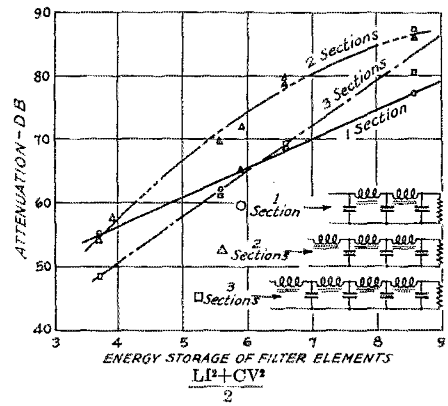


FIG. 7—FILTER ATTENUATION WITH DIFFERENT NUMBERS OF SECTIONS FOR VARIOUS VALUES OF ENERGY STORAGE

Data from Table V

MULTI-SECTION FILTERS

Filters consisting of a first choke followed by a single section give entirely adequate smoothing for the majority of amateur needs. When exceptional ripple attenuation is desired, however, the addition of further sections often may be preferable to adding to the size of units in the more standard type. In other words, if the units are so large that the resonance points are very far below the operating frequencies, more attenuation will be obtained by adding sections of smaller units, even though the resonance points are raised. A rough and ready rule for the best number of sections is given by the following formula:

$$n = \sqrt{\frac{\omega^2 LC}{16}}$$

Where L is total inductance, *excluding* the first choke, in henrys, C is total capacity in farads, ω^2 is $(2\pi f)^2 = 0.57 \times 10^6$ for 120 cycles.

As this formula involves some very rough assumptions and disregards losses in the filters, it usually gives slightly too many sections; but it does serve as a guide as to whether or not it would be advisable to break up the filter into more sections.

DISCUSSION OF TEST RESULTS, TABLE III AND CURVES OF FIG. 5

The tests covered by all the tables were carried out as described above following the circuit of Fig. 2. The first columns cover the values of L

TABLE III

SMOOTHING WITH CONSTANT CAPACITY AND VARIABLE INDUCTANCE

L_1 = 12 henrys $C_1 = C_2$ = 2 & 4 μfd.
L_2 = variable R = 2870 ohms

	L_2	C_1	C_2	*Volts Meas.*	*Ripple Calc.*	Z_r	*Attenuation Ratio*	*D.B.*	*% Ripple*	$\frac{LI^2+CV^2}{2}$
1. a.	7.5	2	2	3.8	3.85	$.336\times10^6$	$.855\times10^{-3}$	21.2	0.44	3.7
b.	15.0	2	2	1.9	1.92	.672	.427	27.2	0.22	4.0
c.	30.0	2	2	0.8	0.95	1.34	.214	33.4	0.09	4.6
d.	45.0	2	2	0.6	0.64	2.02	.142	36.8	0.07	5.3
2. a.	15.0	4	4	0.45	0.48	2.68×10^6	.107	39.4	0.052	6.2
b.	30.0	4	4	0.24	0.24	5.36	.535	45.5	0.028	6.8
c.	45.0	4	4	0.17	0.16	8.04	.357	49.0	0.020	7.5

TABLE IV

SMOOTHING WITH CONSTANT INDUCTANCE AND VARIABLE CAPACITY

L_1 = 12 henrys C_1 & C_2 variable
L_2 = 15 – 30 – 45 henrys R = 2870 ohms

	L_2 *h*	C_1 *μfd.*	C_2 *μfd.*	*Total C μfd.*	*Volts Meas.*	*Ripple Calc.*	Z_r *Ohms*	*Att. Ratio*	*D.B.*	*% Ripple*	$\frac{LI^2+CV^2}{2}$
1. a.	15	1	1	2	7.0	7.4	$.168\times10^6$	17.2×10^{-3}	15.4	0.81	2.3
2. a.	15	1	2	3	3.5	3.3	.378	7.6	22.8	0.41	2.9
b.	15	2	1	3	3.5					0.41	
3. a.	15	2	2	4	1.9	1.8	.672	4.3	27.2	0.22	3.4
4. a.	15	2	4	6	0.71	0.82	1.51	1.9	34.6	0.095	4.5
b.	15	4	2	6	0.71					0.095	
5. a.	15	4	4	8	0.45	0.45	2.69	1.1	39.4	0.052	5.6
6. a.	30	2	1	3	1.45	1.71	$.75\times10^6$	3.8×10^{-3}	28.4	0.170	3.6
b.	30	1	2	3	1.45					0.170	
7. a.	30	2	2	4	0.80	0.95	1.33	2.16	33.4	0.093	4.1
8. a.	30	4	4	8	0.24	0.23	5.32	0.54	45.4	0.028	6.3
9. a.	45	2	2	4	0.60	0.64	2.01×10^6	1.43×10^{-3}	36.8	0.07	4.8
10. a.	45	2	4	6	0.35					0.041	
b.	45	4	2	6	0.30	0.29	4.53	0.63	44.0	0.035	5.9
c.	45	3	3	6	0.27					0.031	
11. a.	45	2	6	8	0.21					0.024	
b.	45	6	2	8	0.22	0.16	8.05	0.36	49.0	0.026	7.0
c.	45	4	4	8	0.17					0.020	
12. a.	45	2	10	12	0.112					0.013	
b.	45	4	8	12	0.068	0.071	18.10	0.16	56.0	0.008	9.1
c.	45	6	6	12	0.061					0.007	

TABLE V

COMPARISON OF SINGLE AND MULTI-SECTION FILTERS

L_1 = 12 henrys $C_1 - C_2$, etc., as given
$L_2 - L_3$, etc., as given R = 2700 to 2900 ohms

No.		*Total L & C*	*Distribution L*	*Distribution C*	*Volts Meas.*	*Ripple Calc.*	Z_r $\times 10^6$	*Atten. Ratio* $\times 10^{-3}$	*D.B.*	*% Ripple*	$\frac{LI^2+CV^2}{2}$‡
1.	a.	26*h.*	26	2–2	0.80	1.10	1.61	1.78	55.0	.093	
	b.	4 μfd.	13–13	1–1–2	0.85	1.50	1.51	1.89	54.4	.099	3.71
	c.	n=1.9*	6.5–6.5–6.5†	1–1–1–1	1.63	16.40	0.79	3.62	48.8	.190	
2.	a.	30*h.*	30	2–2	0.80	0.95	1.61	1:78	55.0	.093	3.89
	b.	4 μfd.	15–15	1–1–2	0.80	1.06	1.61	1.78	55.0	.093	
		n=2.1									
3.	a.	30*h.*	30	4–4	0.24	0.24	5.3	.535	65.4	.028	5.89
	b.	8 μfd.	15–15	2–2–4	0.11	0.11	12.0	.245	72.2	.013	
		n=2.9									
4.	a.	45*h.*	45	3–3	0.350	0.285	3.7	.780	62.0	.041	
	b.	6 μfd.	30–15	2–1–3	0.144	0.112	9.0	.320	69.8	.017	5.56
	c.	n=3.1	15–15–15	2–1–1–2	0.380	0.014	3.4	.850	61.4	.044	
5.	a.	45*h.*	45	4–4	0.165	0.160	7.8	.370	68.6	.019	
	b.	6 μfd.	15–30	2–2–4	0.048	0.044	27.0	.107	79.4	.006	6.56
	c.	n=3.6	30–15	2–2–4	0.051	0.044	25.3	.114	78.8	.006	
	d.		15–15–15	2–2–2–2	0.098	0.018	13.2	.218	69.2	.011	
6.	a.	45*h.*	45	6–6	0.061	0.071	21.1	.135	77.4	.007	
	b.	12 μfd.	30–15	4–4–4	0.021	0.013	61.4	.047	86.6	.025	
	c.	n=4.4	15–30	4–4–4	0.022	0.013	58.6	.049	86.2	.025	8.56
	d.		15–15–15	2–2–4–4	0.020	0.008	64.5	.045	87.0	.023	
	e.		15–15–15	2–2–2–6	0.041	0.008	31.4	.091	80.8	.048	

* "Best" number of sections from: $n = \sqrt{\frac{a^2LC}{16}}$

† 6.5 henrys added to L_1 to avoid resonance.

‡ $\frac{LI^2+CV^2}{2}$ includes 12 henrys for first choke.

and C used. The calculated ripple comes out almost exactly the same as the measured value. The percent ripple is obtained by dividing the observed value by the terminal voltage of 860. The extreme right-hand column gives the total size of filter units in terms of watt-seconds storage. Plotting the observed values of ripple and size of second choke in Fig. 5a shows the relation between the two. This curve can be used directly to obtain the desired sizes of choke and condenser within its range for any voltage operating into the same load resistance, since the ripple is calculated for almost exactly 1000 volts per transformer side. Any other voltage would produce proportional ripple. Furthermore, the results can be used roughly for other values of resistance load, since only very wide departures from test conditions change the ripple, *provided that the true inductance is known for the current resulting from the proposed voltage and resistance.*

Curves in Fig. 5b are more interesting from an engineering standpoint, but are more difficult to visualize and require calculation for use. The d.b. attenuation is plotted against the total energy storage of the filter system. It will be noted that for each value of capacity, changes in inductance make two curves. The line A–B is drawn tangent to these two curves. Some other value, say 3 μfd. in each leg, would give another curve lying between the two plotted and tangent to the line A–B at some intermediate point, such as 6 watt-seconds. Therefore it is evident that *for each value of shunt capacity there is some value of inductance that will give more attenuation than any other value.* Taking the two tangent points in Fig. 5b and interpolating constants, we find that these correspond to a value of $\sqrt{\frac{L}{C}}$, substantially equal to the load resistance. This conforms to the condition of equal energy storage in chokes and condensers discussed above and the desirability of this relation is thus shown empirically by the tests.

DISCUSSION OF TEST RESULTS, TABLE IV AND CURVES OF FIG. 6

These tests were very similar to those covered under Table III except that the inductance was held constant and the capacity varied. Three values of inductance were used. In each group the distribution of the condensers was varied and it will be seen that little or no change in ripple resulted. For other reasons, best operation usually will be obtained with the larger condenser in the last position. For example, in group 11, a, b and c, the 8-μfd. condenser was split up with 2–6, 6–2 and 4–4 μfd. Very little difference in ripple was observed for the three combinations. Even distribution of capacity gives slightly the better results, and the value checking closest with the calculated value. As filter design calls for uniform distribution of condensers in the formulas used, this is of course to be expected. The greatest difference due to non-uniform distribution occurs in group 12, line *a*, where 12 μfd. distributed as 2–10 gives almost double the ripple obtained with equal capacity in each leg. With this amount of energy storage, however, the ripple is so small that either one would be very satisfactory for almost any power supply which did not have great subsequent amplification.

The curves of Fig. 6a are plotted in similar manner to those of Fig. 5a and are self-evident. The same remarks hold as to their availability for interpreting filters used with other power combinations. The curves of Fig. 6b show the same type of curvature as before, although it is not so pronounced. The line A–B was drawn as before and, estimating points of tangency, we find that the best filter combinations for maximum attenuation with minimum material occur when $\sqrt{\frac{L}{C}}$ is close to the value of the load resistance. This point is not very critical and considerations of cost might result in some other combination being cheaper or lighter. Therefore starting with the inductance-capacity ratio as a first criterion, the particular point it is desired to emphasize can be studied. If energy storage is greater than 4 or 5 watt-seconds it is probably better to go to a multi-section filter anyway.

The calculation of voltage ripple for multi-section filters becomes more difficult, the approximate formula being given below. Small errors in the values of inductance are greatly multiplied by the term raised to the power of the number of sections. For example if: ω^2LC is about 4, subtracting 2 from it and then cubing, for a three section filter, obviously will give a result very wide of the mark unless the constants are very closely known. Therefore such calculations will give only the magnitude of results, but are useful as guides. More accurate calculation can be obtained with more detailed formulas and by taking into account the losses in the filter circuit; but this becomes rather involved and much too complicated to consider here.

The formula for calculating Z_r is

$$Z_r = \frac{\omega^2 R L_1 C}{2} (\omega^2 L_n C - 2)^n.$$

The terms are to be evaluated as follows:

$\omega^2 = (2\pi f)^2$ or 0.57×10^6 for 120 cycles.

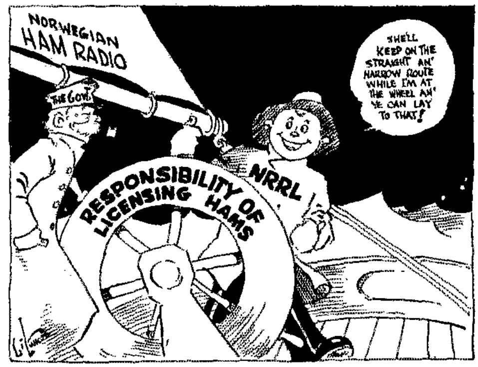
R is the load resistance in ohms.

L_1 is the inductance of the first choke in henrys (12 henrys).

C is the capacity per section, which is taken as the total capacity divided by $(n+1)$ when not equally distributed.

(*Continued on page 88*)

in getting certain of them condemned, and this it is our intention to continue. This policy, which is obviously radically different from that which certain other European societies must follow, may probably explain to some Union members our absolute refusal to support illicit Norwegian stations, forward cards, etc. We may in this



connection point out that the official list of calls is published in each issue of the "Call Book," and although additions may occur in the interval between issues, this will in any case give a quite good record of our licensed stations.

Our immediate expectations include the printing of our "Bulletin," as already mentioned; the erection of a Headquarters transmitter; the organization of an inland relay net encircling all the country; and the better coöperation between I.A.R.U. sections, especially in Europe. Regarding Madrid, we are keeping careful watch on the position of our own Government officials, we are hopeful as regards the rest, and we expect great things from our untiring Union Secretary, Mr. Warner.

The Economical Design of Smoothing Filters

(Continued from page 40)

L_h is the inductance per section after the first choke, and is taken as the average value if the various chokes are not identical.

n is the number of sections, counted for the groups after the first choke. Thus two chokes and two condensers in the usual arrangement are counted as one section, another choke and condenser added on are two sections, etc.

The results of various combinations are tabulated in Table V and plotted graphically in Figure 7. The various calculated values can all be obtained from the value of Z_r as for single sections.

DISCUSSION OF TEST RESULTS, TABLE V AND CURVES OF FIG. 7

Table V shows the results of a large number of combinations of inductance and capacity in various numbers of sections. In each group the total inductance and capacity is held constant and its distribution varied. These combinations are