exponentially to zero. This limiting case corresponds to that discussed by Rayleigh.[5] The decreasing exponential is characterized by an attenuation constant, and since the current density is alternating and the phase changes linearly with distance, there is also a phase constant. Expressed respectively as the distance required for the magnitude to fall to $1/e$ of its value at the surface, and as the distance for the phase of the current density to change by 1 radian, the two constants are equal and given by $b/\sqrt{2}$. The reciprocal $\sqrt{2}/b$ is the "skin thickness" or depth of penetration[6,7] and for copper at 20 degrees centigrade has the value $2.64/\sqrt{f}$ inches. At a depth equal to three skin thicknesses, the current density has a value 5 per cent of that at the surface and lags by 3 radians or nearly 180 degrees.

In the limiting high-frequency case, the total root-mean-square current, which is the integral of current density over the cross section with due regard to phase, approaches in magnitude the product $J_s\delta p/\sqrt{2}$, where J_s is the root-mean-square magnitude of surface current density, p the perimeter, and δ the skin thickness, all in m.k.s. units. As the frequency increases without limit, the phase of the total current approaches an angle of 45 degrees lagging behind the phase of the surface current density. This phase relationship of 45 degrees means that the resistance per unit length approaches equality with the internal reactance per unit length. This equality together with the information in Fig. 1 permits a rapid determination of the internal inductance of copper-covered steel when the frequencies are high enough so that the limiting case holds.

The resistance per unit length of the conductor in the limiting range is simply the quotient of resistivity by skin area or $\rho/\delta p$; that is, the resistance is the direct-current resistance of the "skin." Thus the concept of skin thickness may be employed to determine the required size of conductor such that the conductor has a given resistance at any specified frequency in the limiting range. When br_2 is 30 or more, the error is 2 per cent or less.

Also, the ratio of the depth of copper in copper-covered steel to skin thickness is a measure of whether there is enough copper to make the greatest possible use of copper to increase the conductance. From Fig. 4 it follows that if br_2 is 14 for the 30 per cent conductor or 8 for the 40 per cent conductor which, respectively, correspond to a copper depth of 1.3 and 1.1 skin thicknesses, the resistance of the copper-covered steel is at least as low as that of a solid-copper conductor of the same diameter. In Fig. 8 the copper depth is 1.4 skin thicknesses.

Acknowledgment

The authors gratefully acknowledge the helpful consultations with R. Selquist and F. E. Leib of the Copperweld Steel Company.

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[6] C. P. Steinmetz, "Transient Electric Phenomena and Oscillations," McGraw-Hill Book Company, New York, New York, 1909.
[7] Harold A. Wheeler, "Formulas for the skin effect," Proc. I.R.E., vol. 30, pp. 412–424; September, 1942. This paper also includes an excellent list of references on skin effect.

Corrective Networks for Feedback Circuits*

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Summary—**Design information is given for simplified corrective networks for application with negative-feedback devices. The corrective networks are applied to control the cutoff attenuation characteristic of the feedback transmission loop to prevent oscillation. To provide a design factor of safety against oscillation, the cutoff transmission characteristic should not attenuate at a rate of more than 10 decibels per octave. Corrective networks are required to achieve this attenuation rate.**

Introduction

THE requirements for stabilizing amplifying devices with negative feedback against oscillation have been given in the technical literature in recent years.[1–4] These requirements relate to certain necessary conditions of phase shift and attenuation in the amplifier and feedback circuits. To obtain an optimum design that satisfies these requirements, it has been shown in previous literature that definite attenuation characteristics must be followed. Networks are given in this article which may be used to obtain amplifying devices that will approach the optimum characteristics. These networks consist of simple circuit-element combinations that are suited for operation in the plate and grid circuits of vacuum-tube amplifiers.

Fig. 1 shows a schematic feedback amplifier with a gain of $\mu\underline{/\theta}$ and a feedback network of $\beta\underline{/\psi}$. The net gain of the amplifier is given by the relation

$$\text{net gain} = \mu\underline{/\theta}\,/(1-\mu\underline{/\theta}\,\beta\underline{/\psi}). \tag{1}$$

In the normal frequency range the phase of $\mu\underline{/\theta}\ \beta\underline{/\psi}$ is adjusted to give a feedback action that opposes the applied signal, thereby reducing the gain and giving negative feedback. The quantity $\mu\underline{/\theta}\ \beta\underline{/\psi}$ is the

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[2] H. W. Bode, "Relations between attenuation and phase in feedback amplifier design," *Bell Sys. Tech. Jour.*, vol. 19, pp. 421–454; July, 1940.
[3] F. E. Terman, "Network theory, filters, and equalizers—Part II," Proc. I.R.E., vol. 31, pp. 235–240; May, 1943.
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transmission characteristic of the "feedback loop" and may be measured by breaking the circuit at X of Fig. 1 and applying a signal at A and comparing it with the signal obtained at B. According to the Nyquist stability criteria the system will not oscillate if the transmission characteristic $\mu\underline{/\theta}\ \beta\underline{/\psi}$ has a gain less than unity when the phase shift reaches 180 degrees.

In the normal frequency range of a feedback device, $\mu\underline{/\theta}\ \beta\underline{/\psi}$ is usually nearly constant. Outside the normal

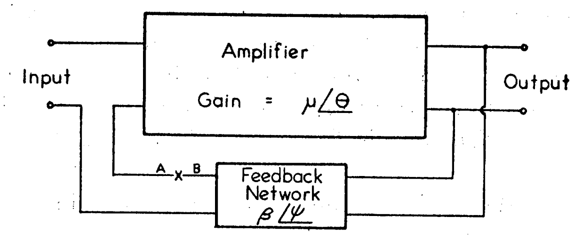

Fig. 1—Schematic of basic feedback amplifier.

frequency range it is desirable to attenuate as rapidly as possible to reduce the gain to less than unity. The only thing that limits a rapid reduction in gain is the phase shift that occurs with the change in gain. The attenuation characteristic outside the normal frequency band must be controlled to prevent the phase shift from exceeding the 180-degree maximum.

Characteristics of Amplifiers

The cutoff characteristic of an amplifier outside of its normal frequency range is determined by the reactive elements of the interstage coupling devices employed. In a resistance-coupled amplifier these usually are stray circuit capacitances and grid-coupling condensers. Each amplifier stage introduces reactive elements which combine to produce a cutoff characteristic that may or may not produce a stable feedback amplifier. The cutoff characteristic usually must be modified to obtain a stable amplifier.

Bode[2] has shown for minimum phase-shift networks that the phase-shift characteristic of a network may be obtained directly from its amplitude characteristic. Thus in the usual amplifier circuits the phase-shift characteristic is directly related to the amplitude characteristic and is independent of the particular circuit combination which produced it. To obtain a special phase-shift characteristic for a feedback amplifier, the corresponding amplitude response needs to be considered.

The usual coupling network with reactive elements has a frequency region for which the amplitude response is uniform as well as a region for which the response varies with frequency. There are certain universal facts associated with these coupling devices operating well into the cutoff region to give an asymptotic response.

1) The amplitude-frequency response is directly or inversely proportional to frequency as the case may be.
2) The phase shift is 90 degrees lagging for the response decreasing with frequency and 90 degrees leading for the response increasing with frequency.
3) With logarithmic co-ordinates the slope is 1 or -1.
4) With logarithmic frequency and decibel amplitude scale the slope is approximately 6 decibels per octave.
5) No network in the asymptotic cutoff region will have a slope of less than unity and all more complicated networks have asymptotic slopes that are integers when plotted on logarithmic co-ordinates.
6) The phase shift is corresponding integral multiples of 90 degrees.

Feedback Requirements

The greatest cutoff attenuation rate that may be employed over any extended frequency spectrum in a feedback device is 12 decibels per octave. This corresponds to a phase shift of 180 degrees which is the limit for stability. In a practical device a margin of safety must be provided, so that a slope of less than 12 decibels per octave must be used. It is customary to allow a 30-degree phase-shift margin, thus giving a slope of $150/180\times 12=10$ decibels per octave.

The usual amplifier has an asymptotic cutoff characteristic which is greater than 10 decibels per octave since each coupling circuit produces 6 decibels per octave or some multiple of 6 decibels per octave. The design of a stable feedback amplifier requires a transition from the region of controlled 10-decibel-per-octave attenuation in which the feedback loop gain is greater than unity,

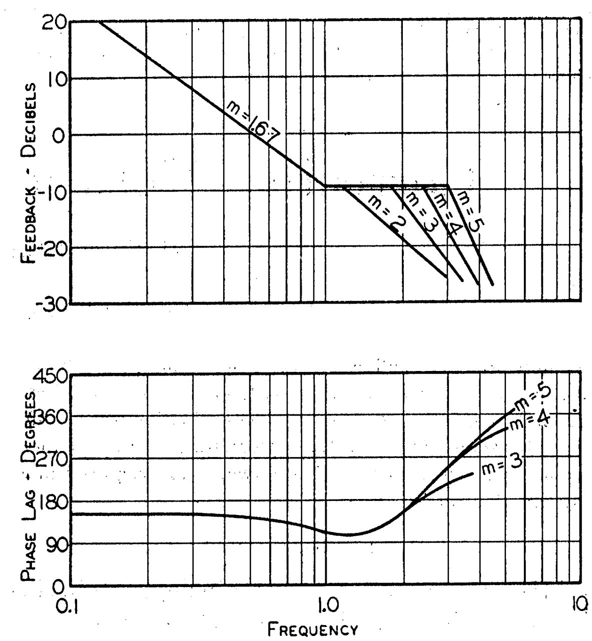

Fig. 2—Ideal amplitude cutoff response with resulting phase shift. m = slope in units of 6 decibels per octave. $m=1.67$ is controlled 10-decibel-per-octave portion of cutoff. $m=2$, 3, 4, or 5 is uncontrolled asymptotic cutoff.

to the region of uncontrolled attenuation in which the attenuation rate may be very high and the loop gain is much less than unity.

This transition may be obtained by following Bode's ideal cutoff transmission characteristic for the "feedback loop" as shown in Fig. 2. The slope of the various segments of the curve are given in units of m which are units of 6 decibels per octave. This cutoff characteristic satisfies Nyquist's stability criteria, since the phase shift is less than 180 degrees for frequencies where the transmission characteristic has a gain greater than unity.

This attenuation curve features a factor of safety against changes in both gain and phase shift. The curve attenuates at a rate of 10 decibels per octave ($m = 1.67$), which provides a phase margin of safety of 30 degrees with respect to the 180-degree maximum.

In addition, a region of zero-gain change is provided to give a phase-shift cancellation effect against the larger phase shift produced by the rapid asymptotic cutoff at the higher frequencies. The frequency range of this step in the attenuation characteristic is equal to the ratio of the slope of the asymptotic characteristic to the slope of the controlled characteristic. The attenuation step is 9 decibels below the unit loop gain (zero-decibel) line, giving a factor of safety against gain variations in the amplifier. The curves illustrated in Fig. 2 apply to the high-frequency cutoff characteristic while similar curves apply equally well to the low-frequency cutoff characteristic.

Modification of the Feedback-Loop Amplitude Response

The cutoff amplitude-response characteristic of most feedback devices will not satisfy the slope requirements

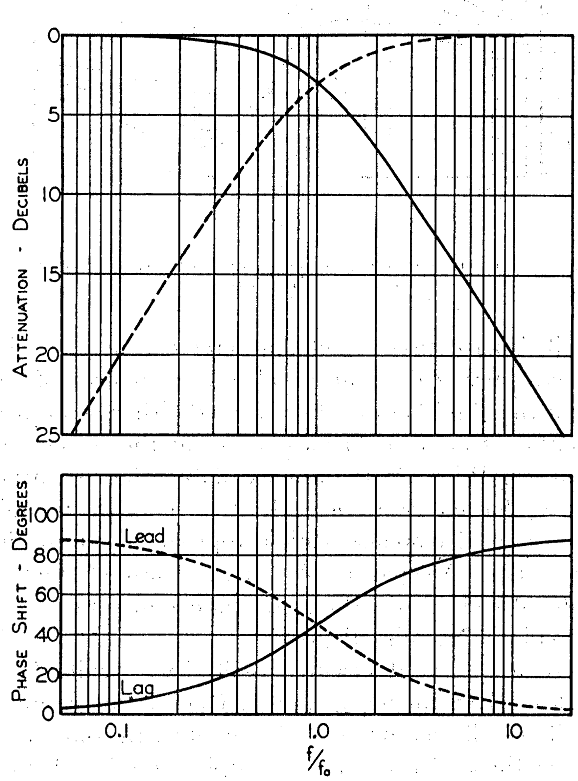

Fig. 3—Transmission characteristic of resistive-reactive coupling elements. $X = R$ at $f/f_0 = 1$.

of the ideal cutoff characteristic. The usual response characteristic cuts off too rapidly or allows no margin of safety. Corrective networks must be added to control the attenuation rate and to provide the necessary step in the attenuation characteristic.

All simple coupling combinations conventionally used with vacuum tubes provide an asymptotic response

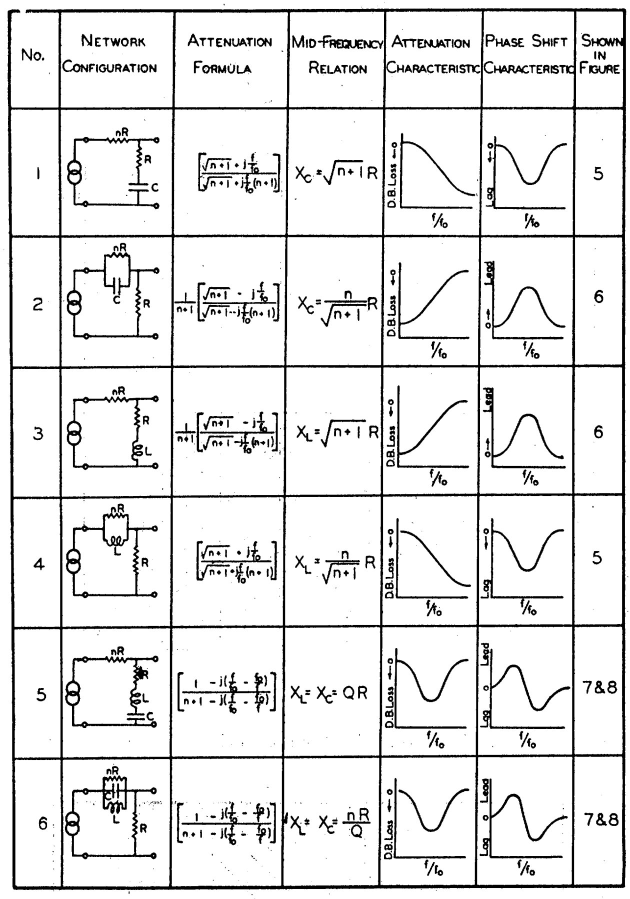

Fig. 4—Design data for corrective networks. Generator is considered to have zero impedance. Load impedance is considered infinite.

characteristic that is a multiple of 6 decibels per octave. For example, the low-frequency response of a transformer-coupled stage will cut off at an asymptotic rate of 6 decibels per octave caused by the shunting effect of the inductance upon the resistive parts of the circuit. At the high frequencies the asymptotic cutoff may be at a rate of 12 decibels per octave due to the combination of leakage inductance and shunt capacitance. Likewise, each stage of resistance coupling gives a 6-decibel-per-octave asymptotic cutoff at the high and low frequencies. Fig. 3 is a plot of the ordinary low- and high-frequency cutoff characteristic for resistance-reactance elements.

To obtain a transmission-slope characteristic of 10 decibels per octave, which is not a multiple of 6 decibels, it is necessary to include corrective networks. To give a net 10-decibel-per-octave slope the corective network may contribute a 4-decibel-per-octave slope to be added to a 6-decibel-per-octave slope, or it may subtract a 2-decibel-per-octave slope from a 12-decibel-per-octave slope. Corrective networks 1, 2, 3, and 4 of Fig. 4 illustrate four different combinations of circuit elements that will give a slope of less than 6 decibels per octave over a limited frequency range. Networks 5 and 6 of

Fig. 4 show two different combinations which may be used to obtain a step in the frequency-response char-

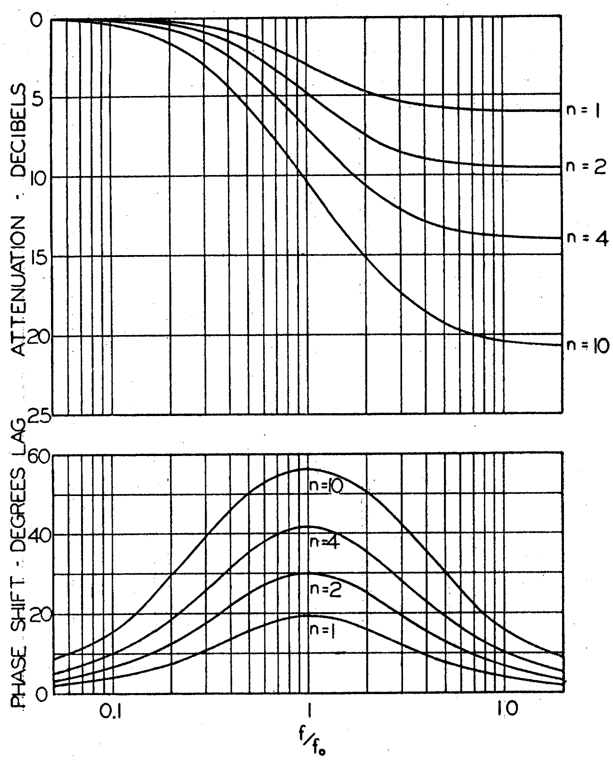

Fig. 5—Response characteristics for corrective networks 1 and 4.

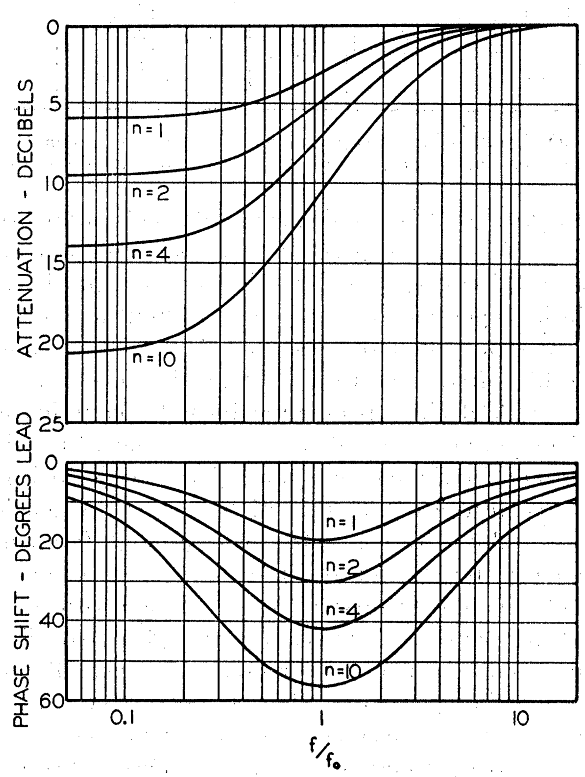

Fig. 6—Response characteristics for corrective networks 2 and 3.

acteristic. The response characteristics of these networks are shown in Figs. 5 to 8.

Each corrective network contains two resistance elements which are related in magnitude by a parameter n. The reactance of the associated capacitance or inductance is obtained by an expression derived for the mid-frequency of the network. Networks 5 and 6 containing both inductance and capacitance are related to the resistive components at the mid-frequency (resonance) by a factor Q. The mid-frequency relation for each corrective network is given in Fig. 4.

Application of Networks

The systematic design of a feedback amplifier using these corrective networks requires the determination of the attenuation or gain that must be introduced. This may be obtained by first plotting in decibels the desired

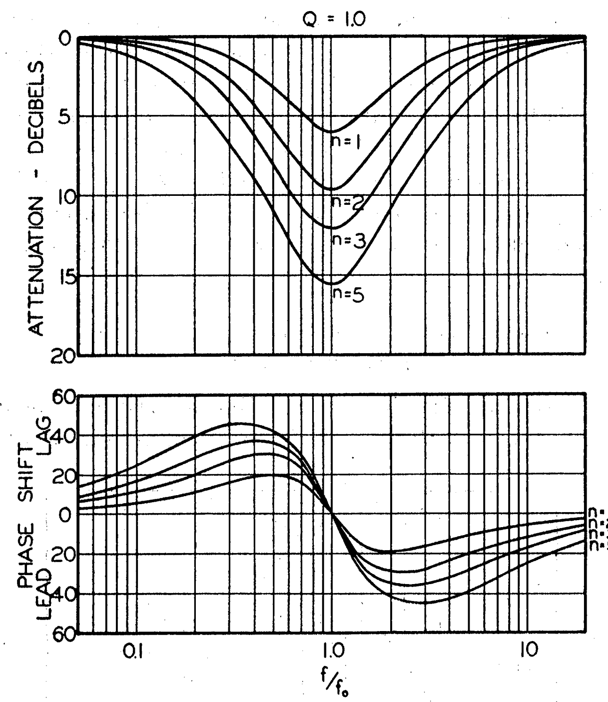

Fig. 7—Response characteristics for corrective networks 5 and 6, $Q=1.0$.

10-decibel-per-octave cutoff characteristic on a logarithmic frequency scale. The amplitude responses of each stage of the amplifier are then plotted either from measurements or from calculations and using the accompanying curves. With these data available the amplitude-response characteristic of each stage may be modified to give a total response which approaches the desired characteristic. A separate plot of the phase characteristics offers a check on the amplitude plot and may be used to plot a Nyquist diagram.

There are several circuit possibilities available for each desired frequency response. One circuit combination will usually require a minimum of extra components by using those already in the coupling device. Two resistive circuit components are inherently available: the grid-return resistor, and the equivalent-generator impedance. These may both be used to advantage in applying the networks of Figs. 4 and 9. Typical vacuum-tube-circuit arrangements with their equivalent circuits are shown in Fig. 9. Once the corrective network circuit configuration is decided upon and its mid-frequency is determined from an amplitude-response plot, sufficient data are available to complete the network design.

Approximations are often useful in simplifying network design. For example, the equivalent-generator impedance is often conveniently considered negligible, or two corrective networks may be used in tandem with the impedance of the first considered negligible with respect to the impedance of the second. Circuit simplification is often possible. For example, a given circuit element may be useful in one network configuration for attenuating at low frequencies and may be useful in another configuration for attenuation at the high frequencies. It is possible to employ one network attenuating over a portion of the frequency spectrum and another network picking up where the first stops. The slope contributed by these networks is governed by two factors: the parameter n, and the frequency separation between the mid-frequency points of each network.

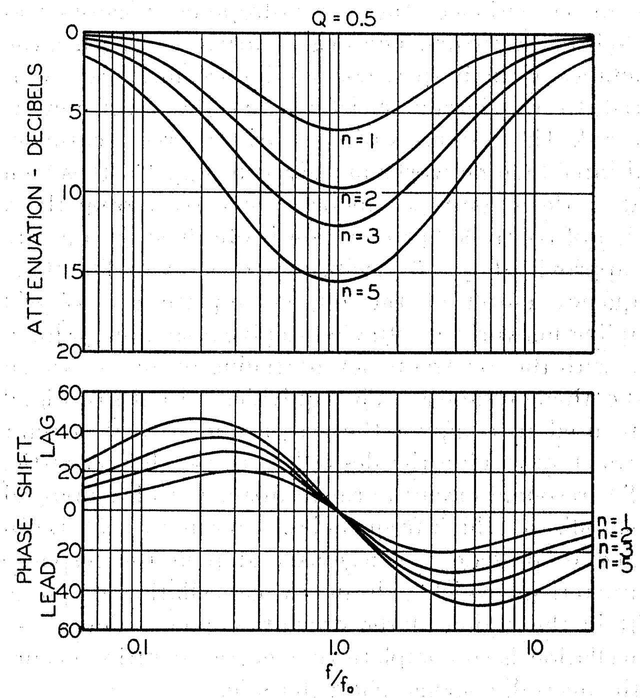

Fig. 8—Response characteristics for corrective networks 5 and 6, $Q=0.5$.

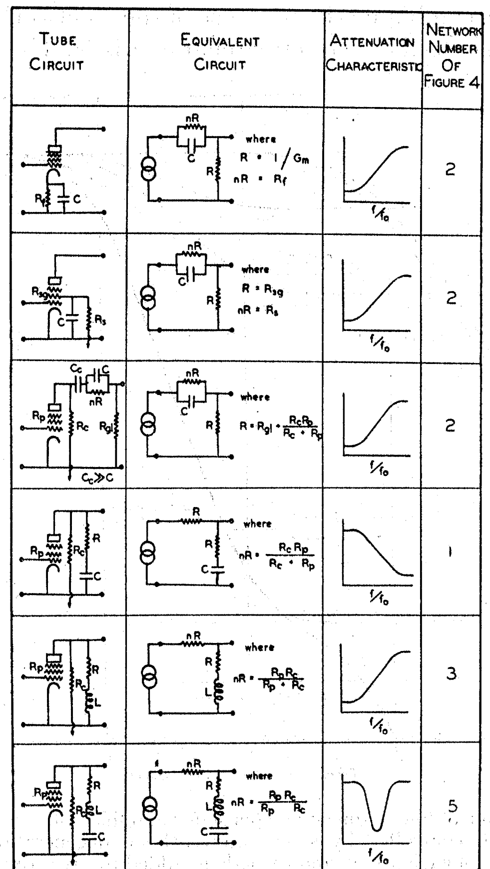

Fig. 9—Typical practical vacuum-tube circuits illustrating use of corrective networks.

Networks 5 and 6 of Fig. 4 are used in the same

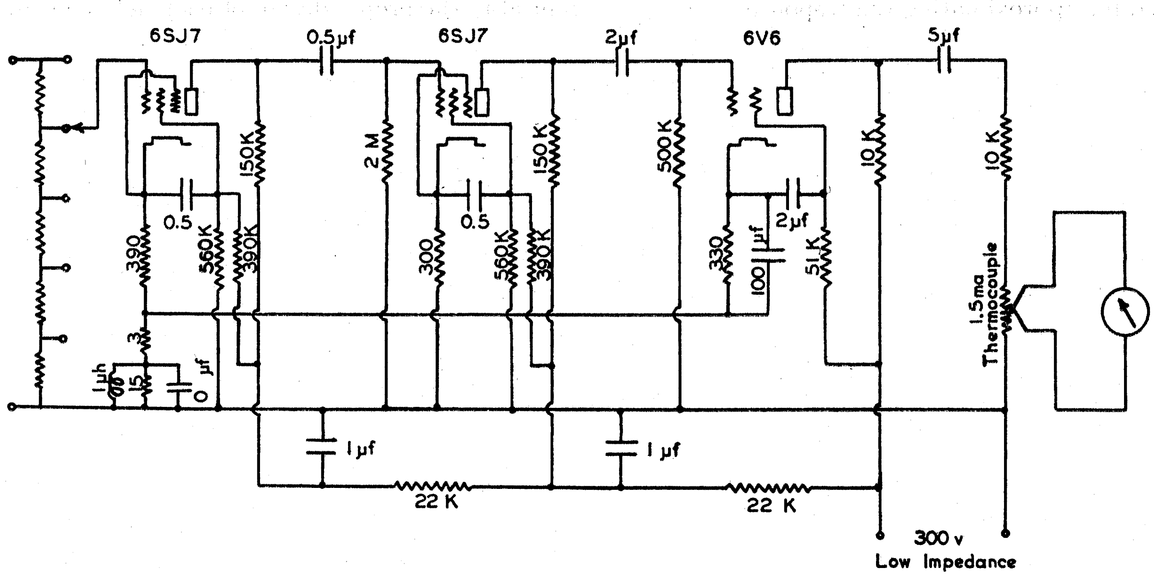

Fig. 10—Circuit diagram of feedback amplifier stabilized by use of corrective networks. R_{sg} for 6SJ7 = 75,000 ohms, $n=3.1$. R_{sg} for 6V6 = 21,000 ohms, $n=2.4$.

manner as the others. They provide a response characteristic which first attenuates and then releases. They are useful in producing the attenuation step of the ideal cutoff characteristic of Fig. 2. The desired step is accomplished by combining the response of these networks with a uniform 6-decibel-per-octave characteristic. The phase-

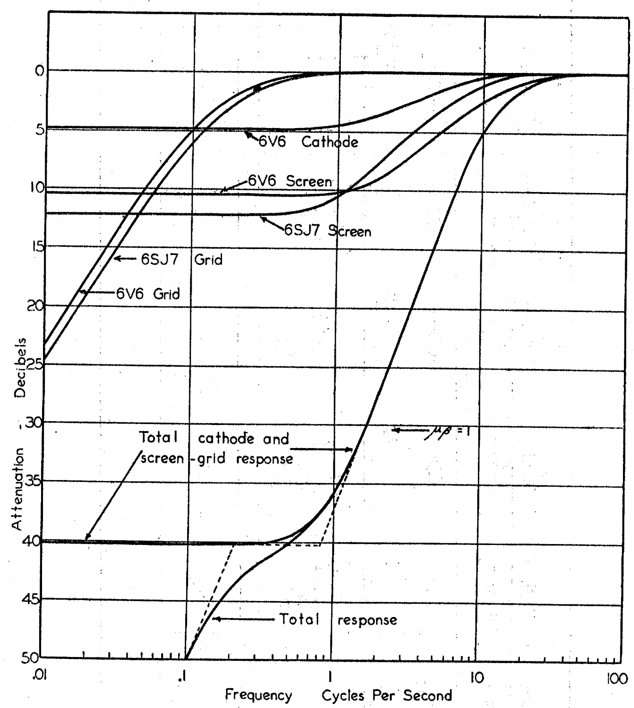

Fig. 11—Plot of low-frequency response for amplifier of Fig. 10. Response is controlled by screen and cathode degeneration. The response of each screen and cathode is added to give step characteristic. Interstage coupling circuits cutoff at lower frequency.

response characteristic for these networks shows a phase-canceling property which helps to avoid oscillation. The selection of the parameters n and Q makes the networks adaptable to various circumstances. There are other resonant configurations for which these curves may be used for approximating the response.

Example

Fig. 10 is the circuit diagram of a feedback amplifier stabilized with the use of corrective network. Negative-current feedback is obtained by producing a voltage in the cathode circuit of the first stage that is proportional to the current in the third stage.

The low-frequency response is corrected by the degenerative action of the screen-dropping resistors and the self-bias resistors. This degeneration occurs when the reactance of the by-pass condensers becomes large compared to the circuit resistance as the frequency is lowered. The responses of the various screen, cathode, and interstage networks are given in Fig. 11. It is seen that if the circuit parameters are chosen properly a region of controlled attenuation is obtained with a rate of approximately 10 decibels per octave. The low-frequency cutoff of the interstage-plate control-grid coupling networks is adjusted to give a step in conjunction with the low-frequency flattening off of the screen and cathode responses. The equivalent circuits of Fig. 9 were used to compute the screen and cathode circuit parameters to give the desired response characteristic.

The resonant circuit in the feedback path is employed to stabilize the high frequencies. A resonant peak is obtained to provide the necessary step in the response characteristic which aids in the cancellation of phase shift in the region of the oscillation point. The phase cancellation is so complete that no regeneration occurs in the over-all response characteristic.

The use of the type of corrective networks shown in this paper offers an approximate method for designing stable feedback devices. Network simplicity is sacrificed for exact response characteristics which is sufficient for many applications. The method of stabilizing feedback devices by obtaining a prescribed amplitude response characteristic is due to H. W. Bode of the American Telephone and Telegraph Company, and an understanding of the results of his work on this subject is quite essential to the proper design of feedback devices.