The Cathamplifier*

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Summary—**This paper discusses an amplifier circuit with high input impedance which permits push-pull operation from an unbalanced source.**

A voltage, proportional to the total circulating current of the push-pull system, is obtained from a transformer connected in the cathode circuits of the output valves. Over-all performance similar to usual push-pull operation may be obtained.

The possibility of a single- or two-mode oscillator is also discussed.

Because of the simplicity, and general application of this circuit, some of the more useful formulas are developed in detail.

I. Basic Theory

In Fig. 1, T is an ideal choke, with an exact center tap, and the whole power due to I_1 I_2 is considered to be developed in the load resister R. In such a case the voltages developed across each half of the winding are equal irrespective of the ratio I_1/I_2.

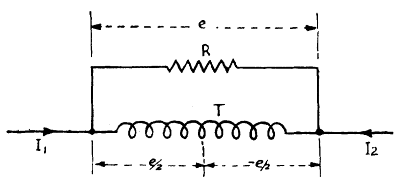

Fig. 1—Basic circuit.

Let R_1 = effective resistance offered current I_1
R_2 = effective resistance offered current I_2.
$I_1/I_2 = Y$.
We have

$$e^2/R = eI_1/2 - eI_2/2$$

$$2e/R = I_1 - I_2 \qquad (1)$$

$$I_1/R_1 = I_2/R_2 = e/2 \qquad (2)$$

$$I_1/I_2 = R_2/R_1 = Y. \qquad (3)$$

Substituting in (1),

$$4/R = 1/R_1 - 1/R_2, \qquad (4)$$

whence

$$R_1 = R(Y-1)/4Y \qquad (5)$$

$$R_2 = R(Y-1)/4. \qquad (6)$$

II. Circuit Application and Basic Conditions for Balance

Fig. 2 shows how the basic circuit is applied to an amplifier in order to obtain push-pull operation. The voltage developed across winding N_3 is applied to the grid of valve 2 in the correct phase, the resister R being varied to obtain ac plate-current balance.

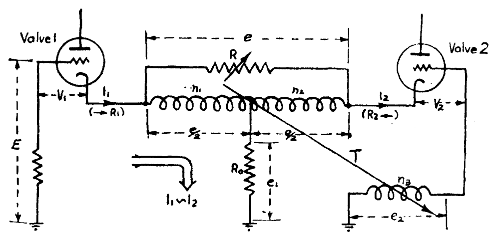

Fig. 2—Basic circuit applied to two valves. An unbalanced voltage E causes push-pull operation by virtue of the transformer T.

Let

$$N_3/(N_1+N_2) = T = e_2/e \qquad (7)$$

G_1 = mutual conductance of valve 1

G_2 = mutual conductance of valve 2.

Also assume negligible distortion and that T is an ideal transformer. In Fig. 2

$$\left.\begin{aligned} I_1 &= V_1G_1 \\ I_2 &= V_2G_2 \end{aligned}\right\} \qquad (8)$$

and

$$V_1G_1/V_2G_2 = Y. \qquad (9)$$

From (2),

$$V_1 \cdot G_1 \cdot R_1 \cdot = V_2G_2R_2 = e/2. \qquad (10)$$

For equilibrium conditions

$$V_2 = e_2 + e_1 - e/2, \qquad (11)$$

where

$$e_1 = (I_1 - I_2)\cdot R_0 \qquad (12)$$

and using (2)

$$e_1 = \frac{e\cdot R_0}{2}\cdot\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$

From (5) and (6), with due regard to the sign of I_2,

$$e_1 = \frac{2e\cdot R_0}{R}\left(\frac{Y-1}{Y+1}\right). \qquad (13)$$

From (7), (11), and (13),

$$V_2 = Te - e/2 + \frac{2eR_0}{R}\left(\frac{Y-1}{Y+1}\right).$$

Simplifying and using (10)

$$2G_2R_1\left(T - 1/2 + \frac{2R_0(Y-1)}{R(Y+1)}\right) = \frac{V_2G_2}{V_1G_1} = \frac{1}{Y}\cdot \qquad (14)$$

* Decimal classification: R363.222. Reprinted from the *Proceedings of the Institution of Radio Engineers, Australia*, vol. II, pp. 199–204, August, 1950, by the courtesy of the Institution of Radio Engineers, Australia.

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If the system is balanced (true push-pull), $Y=1$ and (14) reduces to

$$T = \frac{1 + G_2 R/2}{G_2 \cdot R \cdot}, \tag{15}$$

which is the required information.

III. Degree of Unbalance for Changes in Balance Conditions

Once T is evaluated, the dependence of Y on circuit variables may be obtained. Extracting Y from (14) and substituting for R_1 from (5), we can write

$$Y = \frac{1 - G_2' \cdot (T - 1/2 - 2R_0/R')R'/2}{G_2' \cdot (T - 1/2 + 2R_0/R')R'/2}, \tag{16}$$

where G_2', R' are the values of G_2, R other than those used to evaluate T[see (15)]. Thus for convenience we can put $G_2'R'/G_2R = K \neq 1$.

Using this relation and substituting (15) in (16),

$$Y = \frac{1 - \dfrac{K \cdot G_2 \cdot R}{2}\left(\dfrac{1 + G_2 \cdot R/2}{G_2 \cdot R} - 1/2 - 2R_0/R'\right)}{\dfrac{KG_2R}{2}\left(\dfrac{1 + G_2R/2}{G_2 \cdot R} - 1/2 + 2R_0/R'\right)}.$$

Simplifying,

$$Y = 1 + \frac{2(1 - K)}{K\left(\dfrac{2R_0G_2R}{R'} + 1\right)}. \tag{17}$$

Thus R_0 helps to reduce unbalance. Generally for the simpler circuits $2R_0G_2 \equiv 1$; and if R is nearly equal to R', (17) becomes approximately

$$Y = 1/K. \tag{18}$$

Equation (17) shows that unbalance is independent of valve 1, a most important consideration. In practice, since R can be small, it can be made quite stable so that unbalance becomes almost entirely due to changes in G_2.

When the signal voltages applied to the grids of the valves are large compared to the bias, changes in mutual conductance occur. Hence, in such cases, changes in balance occur with changes in signal level, even though $G_1 = G_2$ at all levels. Equation (17) shows to what extent this may be minimized by R_0.

IV. Effect of Unbalance on Power Output

Power output is affected by lack of balance between the two plate currents. The factors to be considered are

1. $I \sin \omega t \neq I_2 \sin (\omega t + \pi)$.
2. $Y \neq 1$, the input signal being varied, if necessary, to obtain maximum undistorted power.
3. Change in sensitivity when $Y \neq 1$, due to the voltage e_1 (see Fig. 2).

Of these, condition 1 is generally negligible if the departure of the currents from opposite phase is less than 10 degrees. Condition 2 is much more important. This applies for circuits (Fig. 2) in which R is so small that $e/2 \ll V_1$, and $e2=0$ (Ro by-passed). Condition 3 is the more general case and, of course, must be estimated in conjunction with the effect of condition 2. Conditions 2 and 3 are best considered separately.

A. Case for Condition 2

In Fig. 3, $Y \neq 1$, $e/2$ is negligible, e_r is zero, and $E \equiv V_1$.

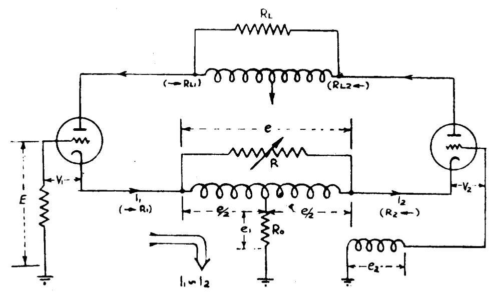

Fig. 3—Complete basic circuit; R_L is the plate load. Power supply circuits are omitted for clarity.

The power in the load R_L may be estimated in the manner used for Fig. 1. Thus let

W_1 = power developed in R_L when $I_1 = I_2$
W_2 = power developed in R_L when $I_1 \neq I_2$
R_{L1} = effective resistance offered I_1
R_{L2} = effective resistance offered I_2.

Then

$$W_1 = (I_1)^2 \cdot R_{L1} + (I_2)^2 R_{L2} \text{ as in equation (1)}$$
$$= (I_1)^2 \cdot R_L \text{ since } I_1 = I_2$$
$$W_2 = (I_1)^2 \cdot R_{L1} + (I_2)^2 R_{L2}$$
$$W_2 = (I_1)^2 \cdot R_{L1}\left(1 + \frac{1}{Y}\right) \text{ from equation (3);}$$

and using (5) (allowing for the sign of I_2),

$$\frac{W_1}{W_2} = \left(\frac{2Y}{Y + 1}\right)^2. \tag{19}$$

Setting a limit of 0.5-decibel loss for the unbalanced condition, the value of Y is found to be 1.13. Using this in (18), the change in G_2R is 11 per cent. As indicated previously, the change may be wholly due to G_2 so that this figure is quite reasonable to maintain. If adjustment of balance is made near maximum output for a particular frequency, (19) shows the power loss resulting from changes in balance at any other frequency.

B. Case for Condition 3

Referring to Fig. 3,

$$E = V_1 + e/2 + (I_1 - I_2) \cdot R_0$$
$$= V_1\left\{1 + G_1 \cdot R_1 + G_1\left(1 - \frac{1}{Y}\right)R_0\right\}$$

by substitution from (3), (8), and (10).

Eliminating R_1 by means of (5) (allowing for the sign of I_2),

$$V_1 \doteq \frac{E}{1 + G_1\left\{\frac{R(Y+1)}{4Y} + \left(\frac{Y-1}{Y}\right)R_0\right\}}. \tag{20}$$

Let V_1' = grid-cathode voltage for the particular case when $Y=1$. Then from (20),

$$V_1' = \frac{E}{1 + G_1 \cdot R/2}, \tag{21}$$

so that using the same nomenclature as before

$$W_1 = (I_1)^2 R_L = (G_1 V_1')^2 R_L. \tag{22}$$

From (19)

$$W_2 = (I_1)^2 \cdot R_L \left(\frac{1+Y}{2Y}\right)^2;$$

and since in this case I_1 is produced by V_1 of (20),

$$W_2 = (G_1 V_1)^2 \cdot R_L \left(\frac{1+Y}{2Y}\right)^2. \tag{23}$$

Substituting (20) and (21) in (22) and (23),

$$\frac{W_1}{W_2} = \left\{\frac{4Y + G_1[R(Y+1) + 4(Y-1)R_0]}{(1+Y)(2+G_1 R)}\right\}^2. \tag{24}$$

For $Y=1.1$, $2R_0G \equiv 1$, $G_1R = 0.4$ (a reasonable figure in practice), $W_1/W_2 = 1.04$ so that the loss is about 0.2 db.

For both cases (A) and (B) above, it was assumed that Y was >1. If this is so, the limit to undistorted output is the drive on valve 1. If we consider that maximum output is obtained when $Y=1$ and that E is constant, then for $Y>1$, V_1 will not be exceeded and distortion due to overdrive will not result. For $Y<1$, I_2 must increase if E is constant. If I_2 increases, V_2 must also increase and overdrive of valve 2 results. At the same time the voltage e_1 causes the voltage V_1 to increase (20) so that overdrive results on valve 1, also. Thus if E is constant, severe distortion will result for $Y<1$. This means that if balance is adjusted at maximum output distortion will occur as R is increased beyond the value required for balance.

From (2) we might expect the power to increase for $Y<1$. But if the power is considered to have a maximum value W_1 when $Y=1$, then distortion results for $Y<1$ and E must decrease so that V_2 becomes the limiting value. Thus the over-all power decreases. It is sufficient for the purpose of this discussion, therefore, that we assume $Y>1$.

V. Inherent Circuit Stability at or Near Balance

There are two possible modes of oscillation:

A. First Mode

Refer to Fig. 3. If I_1 is very small, the phasing of T is such that it would appear oscillation could occur due to feedback in valve 2. If $E=0$, oscillation just commences when

$$e_2 = V_2 + e/2 + (I_2 - I_1)R_0, \tag{25}$$

where $I_2 > I_1$ because valve 2 initiates the condition. Assume this condition arises by varying R to some new value R'. Then R is the value of resistance required for balance and R' is value at which oscillation just commences. From (7), (8), (10), and (25)

$$Te = V_2 + V_2 \cdot G_2 \cdot R_2 + V_2 \cdot G_2 \cdot R_0(1 - Y).$$

Thus

$$2TG_2R_2 = 1 + G_2R_2 + G_2 \cdot R_0(1 - Y).$$

Using (15) and (6) (allowing for the sign of I_2),

$$\left(\frac{1 + G_2R/2}{R}\right)\frac{R'(Y+1)}{2} = 1 + G_2\left\{\frac{R'(Y+1)}{2} + R_0(1-Y)\right\}. \tag{26}$$

Now the current I_1 in this case is produced by the voltages $e/2$ and e_1. Hence,

$$\begin{aligned} I_1 &= G_1(e_1 - e/2) \\ &= G_1[G_2 \cdot V_2(1-Y) \cdot R_0 - G_2 \cdot V_2 \cdot R_2] \end{aligned}$$

from (10), (3), and (12).

In this case R_2 is a component of R', and we can eliminate R_2 by means of (6).

$$\frac{I_1}{G_2V_2} = G_1\left\{(1-Y)R_0 - \frac{R'(1+Y)}{4}\right\}$$

$$Y = \frac{G_1(4R_0 - R')}{4 + G_1(4R_0 + R')}. \tag{27}$$

Substituting for Y in (26), we have

$$\frac{2(1 + G_2R/2)(R'(1 + 2G_1 \cdot R_0)}{R} = 4 + G_1(4R_0 + R') + G_2[R'(1 + 2G_1R_0) + R_0(4 + 2G_1R')];$$

and when $G_1 = G_2$, this simplifies to

$$R' = \frac{2R}{1 - G_1 \cdot R/2} \tag{28}$$

$$= R(2T - 1)/(T - 1) \text{ by using (15).} \tag{29}$$

Perfect stability results when $G_1R = 2(T=1)$. The value of R_0 does not affect the stability. Note that it is impossible for oscillation to occur at or near balance. For if $R=R'$, (29) gives $-G_1R = 2$, which is impossible. If G_1R is negligibly small, then $R' = 2R$, which is well removed from R. If valve 1 is removed, $Y=0$; and from (26),

$$R' = 2R(1 + G_2 \cdot R_0). \tag{30}$$

Again, in the limiting case when $R_0=0$, $R'=2R$, which is a satisfactory value.

B. Second Mode

Fig. 4 shows diagrammatically the equivalent circuit at a high frequency. The grid circuit will consist of the

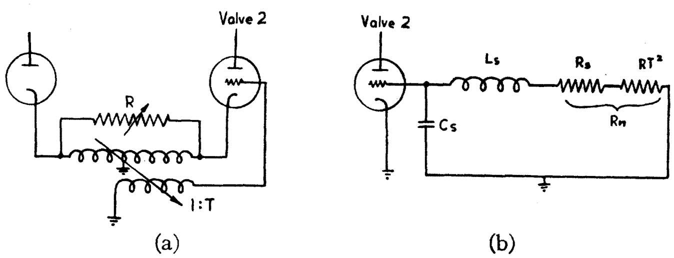

Fig. 4—The equivalent of circuit (a) at high frequencies is shown in (b). *Cs* is stray capacity, *Ls* leakage inductance, and *Rs* secondary resistance.

network comprising leakage inductance *Ls*, self-capacity *Cs*, and effective secondary resistance *Rs*.

$$R_n = (Rs + T^2R),$$

where *Rs* is the secondary resistance. The impedance of this circuit near resonance is $Ls/Cs \cdot Rn$ so that if *Rn* is sufficiently small and *Ls* large a high impedance may result. Feedback from other parts of the circuit, principally the anodes, can, therefore, produce oscillation. Since $Ls/Cs \cdot Rn$ will be a maximum when $R=0$, oscillation may occur if R is too small. Generally, R is sufficiently large to eliminate this problem. As an alternative, the balance adjustment may be accomplished by means of a variable resistance across the secondary of T instead of across the primary.

The control R may, therefore, be used to control oscillation in either mode since for mode A. the value must be large while for B. it must be small; thus oscillation can be made to occur on either side of the balance adjustment. R may also be used as a control for a highly selective amplifier circuit.

VI. Loss of Sensitivity at Balance Due to Degeneration

Referring to Fig. 2, it is seen that a loss of sensitivity occurs due to $e/2$. From (21)

$$E = V_1'(1 + G_1 \cdot R/2)$$

so that the sensitivity is reduced by the factor

$$(1 + G_1 \cdot R/2).$$

By substitution from (15), the loss factor may also be expressed as

$$T/(T-1/2). \tag{31}$$

VII. Balance for Noises Originating in the Power Supply

Fig. 5 is the circuit for examination of this aspect of operation. Let

R_{p1} = plate resistance valve 1
R_{p2} = plate resistance valve 2
E_n = noise voltage in high-voltage supply.

We have

$$V_1 = -(e/2 + e_r)$$
$$V_2 = Te - (e/2 + e_r)$$
$$I_1 = E_n/R_{p1} - G_1(e/2 + e_r) \tag{32}$$
$$I_2 = E_n/R_{p2} - G_2(e/2 + e_r) + TeG_2. \tag{33}$$

From (2) and (6)

$$Te = 1/2 \cdot [TI_2R(Y - 1)].$$

If $R_{p1}=R_{p2}$ and $G_1=G_2$,

$$I_1 - I_2 = -1/2 \cdot [TI_2RG_2(Y - 1)]$$

from (32) and (33). Hence,

$$Y - 1 = 1/2 \cdot [TR(1 - Y) - G_2] \tag{34}$$

for which $Y=1$.

We would expect balance to be independent of the adjustment of R. Balance does depend, however, on the operating characteristics of the valves so that some further control (such as bias or slope) may be provided to adjust this. A simple test of this adjustment is possible with the circuit. For at balance $Te=0$, and no voltage is developed across the secondary of T; thus it is sufficient to observe the grid voltage of valve 2.

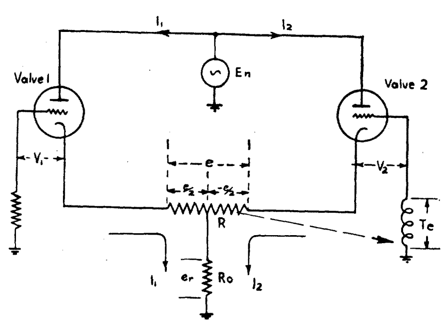

Fig. 5—Equivalent circuit for the examination of noise voltage E_n originating in the high-tension power supply.

VIII. Distortion

The ac plate currents of the valves may be expressed by the usual series,

$$I_1 = aV_1 + bV_1^2 + cV_1^3 + \tag{35}$$
$$I_2 = -a'V_2 + b'V_2^2 - c'V_2^3 + . \tag{36}$$

The total circulating current is the difference between these currents. Usually it is assumed that the coefficients of the two valves are equal ($a=a'$, $b=b'$, and so on), and this will be accepted in the following analysis:

From (35) and (36) the current for any particular coefficient may be expressed in the form

$$I_{n1} = WV_1^n \text{ } n\text{th component for valve 1}$$
$$I_{n2} = W(-V_2)^n \text{ } n\text{th component for valve 2.}$$

The total circulating current for any particular coefficient will depend on the feedback voltages e, e_r, and Te (Fig. 6), occurring for that particular component.

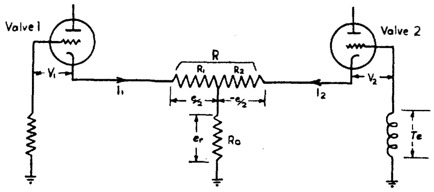

Fig. 6—Basic circuit used for discussion on distortion.

Thus

$$I_{n1} = WV_1^n - (e/2 + e_r)a$$
$$I_{n2} = W(-V_2)^n + (-1)^n(-e/2 + e_r + Te)a$$

since coefficients beyond a for the feedback voltages are negligible. Even components cancel out as with normal push-pull; and for the odd components we have

$$I_{n1} - I_{n2} = W(V_1^n - (-V_2)^n - ea + Tea, \quad (37)$$

where n is an odd number. From (1)

$$e = (I_{n1} - I_{n2})R/2. \quad (38)$$

Substituting in (37) and using (15), we obtain

$$I_{n1} - I_{n2} = W\{V_1^n - (-V_2)^n\} \frac{(2T-1)}{T} \quad (39)$$

Since the distortion in the assumed case is entirely due to odd components, the total distortion is reduced by the factor $T/(2T-1)$. Thus distortion reduction is half the gain reduction (31).

IX. Choice of Balance Resistor and Signal Level for Adjustment of AC Plate Currents

A. Balance Resistor

The ultimate value for R is a compromise. The requirements are perhaps best examined by expressing G_1R in terms of T in the various equations. For maximum possible stability (29) $T=1$; the loss factor (31) is then 2, and the distortion reduction factor (39) is 1. If minimum ac plate resistance is required, then R must be small, which requires $T>1$. Again if R is too small, there is a greater tendency for oscillation in mode 2 unless other precautions are taken. It would appear that the ultimate choice is for a value of T very close to 1.

B. Signal Level

From (18), $Y \propto (1/K)$; from (16), $K \propto G_2'/G_2$. Since $G_2 \propto E$, it follows that $Y \propto E$ (Fig. 2). Thus to obtain maximum power [(24) $Y=1$], balance adjustments should be made at the maximum value of E. However, even harmonics are only cancelled if $V_1 = V_2$ (37). If $a \neq a'$, then when $Y=1$, $V_1 \neq V_2$. If a bias or slope adjustment is also provided, it would appear desirable to adjust at maximum output so that $a=a'$. This results in maximum output and minimum distortion. However, it is sometimes desirable to adjust this extra balance when $E=0$ so that $a=a'$ at low signal level, and minimum noise from the high-tension supply at zero signal is obtained. If this is so, then at the maximum value of E it is unlikely that $a=a'$. The balance adjustment R would then be made to obtain minimum distortion at maximum possible output. Since this would occur when $V_1 = V$, if $a \neq a'$, $Y \neq 1$, and the maximum possible output would be slightly less than obtained with $Y=1$.

X. Conclusion

The circuit under discussion has interesting possibilities. Push-pull operation is possible, yet the input impedance of the circuit is high. Thus the output stages may be preceded by a high-gain voltage amplifier, permitting an extremely compact amplifier unit. This also simplifies the problem of applying inverse feedback over the whole amplifier. The adjustment required for push-pull operation is quite simple and the stability of balance, as well as the inherent circuit stability, is quite good. Cancellation of noise voltages from the power supply is unaffected by the balance adjustment. Slight improvement of distortion may be obtained without excessive loss of sensitivity. It can be expected that changes in balance will occur with signal level, but this should not be any great disadvantage. The circuit permits an easy observation on the adjustment of the operating characteristic when this is provided.

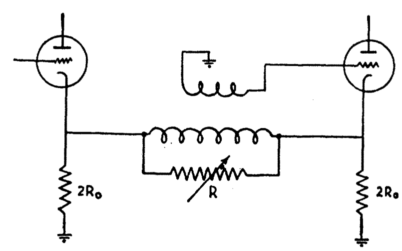

Fig. 7—An interesting modification.

Two modes of oscillation which are independently adjustable are possible. This indicates that the circuit could be used as a highly selective amplifier incorporating a variable selectivity control. Fig. 7 is an interesting

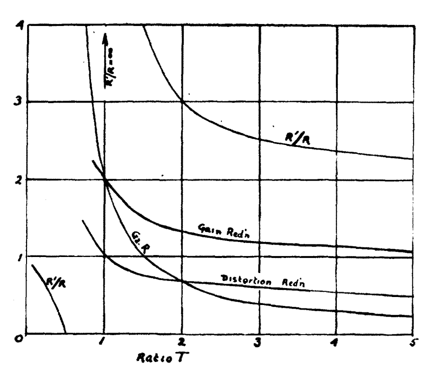

Fig. 8—Showing the relation between the various formulas, and the transformation-ratio T.

modification of the basic circuit discussed above; it does not require a center tapped transformer.

Fig. 8 shows the values of various factors in the foregoing formulas, plotted against the transformation ratio T. (See preceding page.)

XI. ACKNOWLEDGMENTS

The author is indebted to Ferguson Transformers Pty. Ltd. for the use of their laboratory in investigating this circuit; also to Mr. R. Edwards, who assisted in the initial draft paper.

Recent Developments in High-Power Klystron Amplifiers*

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Summary—**The present status of klystron power amplifiers is reviewed by discussing the three types of electron-beam focusing used. The maximum power output, efficiency, gain, bandwidth, temperature compensation, and tuning means of present-day klystrons are briefly discussed. Examples of several typical tubes are given.**

I. INTRODUCTION

THE INCREASING development effort devoted to high-power klystrons, as contrasted to milliwatt local-oscillator types, has brought about a rather rapid improvement in the characteristics of transmitter-type klystrons. There has been an increase in power output and efficiency, large power gain has proven to be very useful, and tuning and temperature compensation characteristics have developed to a satisfactory point. The development of tubes for pulsed applications has opened up this particular field to high-stability frequency control. The increasing use of the radio spectrum is making good frequency control important for both continuous-wave and pulsed operation.

II. TUBE CHARACTERISTICS

1. *Types of Focusing*

First, the capabilities and characteristics of present-day transmitter-type klystrons will be discussed by dividing them into three groups corresponding to the type of electron-beam focusing used. Tubes both with and without grids at the interaction gaps will be discussed.

An electrostatic-type electron gun has been used to form the electron beam of virtually all present day klystrons, but the high-density beams utilized have been maintained over a long path in three different ways, namely: ion focusing, space-charge focusing, and magnetic focusing.

In the first case, positive ions formed from the residual gas molecules neutralize the space charge of the electrons in the beam, and thus allow the beam to continue without space-charge spread. Ion focusing is, of necessity, limited to cw tubes since no ions would be present at the beginning of a pulse to focus the beam. Ion focusing adapts itself most readily to klystrons having grids at the interaction gaps since it is limited to larger diameter beams which are best coupled to the rf voltage by a grid structure.

In the case of space-charge focusing, the electron beam is allowed to follow a trajectory determined by the initial convergent angle of the electron gun and by the mutual space-charge repulsion forces of the electrons. The beam comes down to a minimum, and then spreads out so that it does not have a uniform diameter over its length. This type of beam fits best into a pulsed tube having gridded interaction gaps.

The third type of beam maintenance is by means of an axial magnetic field as described by Wang.[1] After the beam is formed by an electron gun, it may be focused for any distance desired by application of the correct value of uniform magnetic field. This type of beam focusing is useful for both cw or pulsed tubes which do not have grids at the gaps.

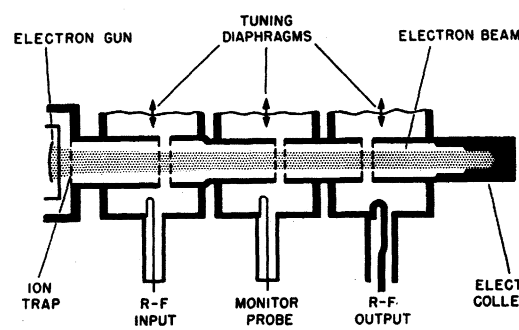

Fig. 1(a)—Example of ion-focused beam in a three-cavity amplifier with gridded gaps. The tube is the SAS-28 with 250-watts output at 2,600 mc and 30 db gain.

Fig. 1 shows examples of these various beam-focusing types. Fig. 1(a) shows the cross section of a three-cavity, ion-focused, gridded, cw tube with 250-watts

* Decimal classification: R355.912.3. Original manuscript received by the Institute, April 30, 1951; revised manuscript received, November 2, 1951.

† Sperry Gyroscope Company, Great Neck, L. I., N. Y.

[1] C. C. Wang, "Electron beams in axially symmetric electric and magnetic fields," PROC. I.R.E., vol. 38, pp. 135–148; February, 1950.